The Concept of Slowing Down to Catch Up with Satellites*

Orbit transfer equations for rendezvousing with a satellite can be very complex. My first job out of college, shortly after sputnik had tempted many of us from more artistic endeavors, involved these ghastly critters. But there are many simplifications that can be made when rendezvous is imminent. In incarnation as a Boeing Aerospace tech staff engineer, I was called upon to review a space robotics project in which complex control laws were being implemented to maneuver within coplanar orbits. The object is to liaison with a previously installed satellite.

Simpler schemes using gravity against itself popped into my head as more appropriate than the complexity that was being employed. Consider, for example, a situation where a robot satellite is in a coincident orbit with the satellite with which it must rendezvous. In this case merely applying an instantaneous impulse to slow the robot down will result in catching up to the satellite it has been following. In consideration of this notion that *slowing down* allows the trailing satellite to *catch up*, there are some interesting observations requiring very little 'rocket science'.

Of course, to begin one must be aware that in a circular orbit the centrifugal and centripetal forces are equal and opposite, so that the velocity of the satellite is inversely proportional to the square root of the orbital radius:

$$|v_c| = K / r^{1/2}$$

where K^2 is the product of the gravitational constant, the mass of the earth and a unit scaling coefficient. The absolute magnitude of the orbital velocity is $|v_c|$ and r is the distance of the orbit from the center of the earth. The mass of the satellite is irrelevant. From this we see that the period is:

$$T_c = 2\pi r / |v_c| = 2 \pi r^{3/2} / K$$

So that satellites in lower orbits, although having lower energy, possess higher velocities and have shorter periods than satellites in higher orbits.

It helps also to notice the obvious, i. e., that at *apogee* and *perigee* in eccentric orbits, a satellite's velocity is horizontal. See for examples, figure 1, for which such orbits are shown in relation to a circular orbit. Notice that the satellite whose orbit is at apogee when it is tangent to the circular orbit has a smaller velocity at that point (but the orbit has a shorter period) than the satellite with the circular orbit. The opposite is the case for the satellite whose perigee happens to be tangent to the circular orbit. Thus, applying horizontal impulses at these points will result in an instantaneous transfer to another orbit that will end up being tangential to the original orbit exactly one period (of the eccentric orbit) later.

To attain a slightly different circular orbit, the following procedure can be applied: For small differences in radii, the impulse Δv that must be applied is approximately half of the difference in the velocities, δv, of the two satellites in the unique circular orbits. Their radii will differ by the amount δr, where:

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$$\delta v = \frac{\partial}{\partial r}[\, K / r^{1/2} \,]\, \delta r = -\, [\, \tfrac{1}{2} K / r^{3/2} \,]\, \delta r$$

Impulses of $\Delta v \cong \tfrac{1}{2}\, \delta v$ must be applied to slow the robot down at both the apogee and perigee of the eccentric transfer orbit. Slowing down twice at these critical points will actually result in the robot attaining a greater velocity than that with which it began. The difference in periods of such impulsively related circular orbits is very nearly:

$$\delta T_c = (\, 3\, \pi / K \,)\, r^{1/2}\, \delta r$$

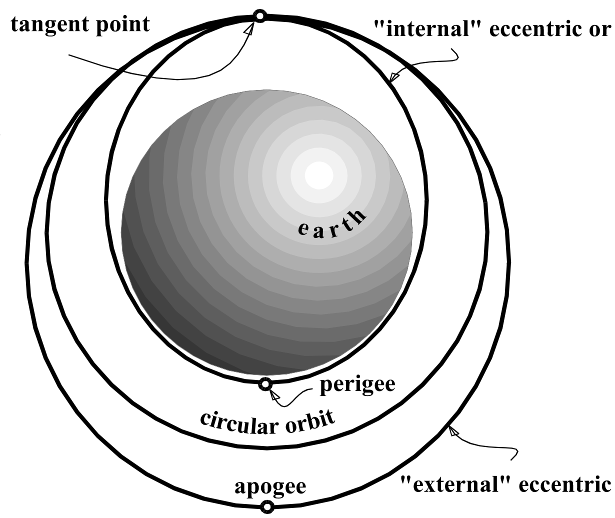


Figure 1: Geometry of circular and eccentric orbits tangent at the points where impulses Δv are to be applied

On an internal orbit achieved by *slowing down*, the robot will pick up *ground* on the satellite it was following either by remaining in that particular eccentric orbit or by subsequently transferring back into the lower circular orbit as shown in figure 2. For an 'external' orbit (also shown), return would be to a point further behind the 'chased' satellite.

It is interesting to view these trajectories in the 'local level' coordinates of the satellite that is being pursued. Figure 2 illustrates various trajectories in this system of reference that differ primarily in the size of the initial impulse, Δv, all of which are negative, i. e., to the right. In the looping path AS, the eccentric orbit obtained by slowing down only slightly is maintained until finally (four orbits later) rendezvous is achieved. In case ABCS, a smaller circular orbit is attained by applying another (also negative) impulse at perigee (B) and proceeding in that lower orbit of a shorter period until another (now positive) impulse sets it back on the eccentric orbit in time for rendezvous. In path ADS an impulse is selected to result in a single eccentric orbit period to achieve rendezvous. In path AEFS the robot overshoots the satellite so a positive impulse (in addition to the one to place it back in the original orbit) is applied to place it in an 'external' eccentric orbit for one period to effect rendezvous.

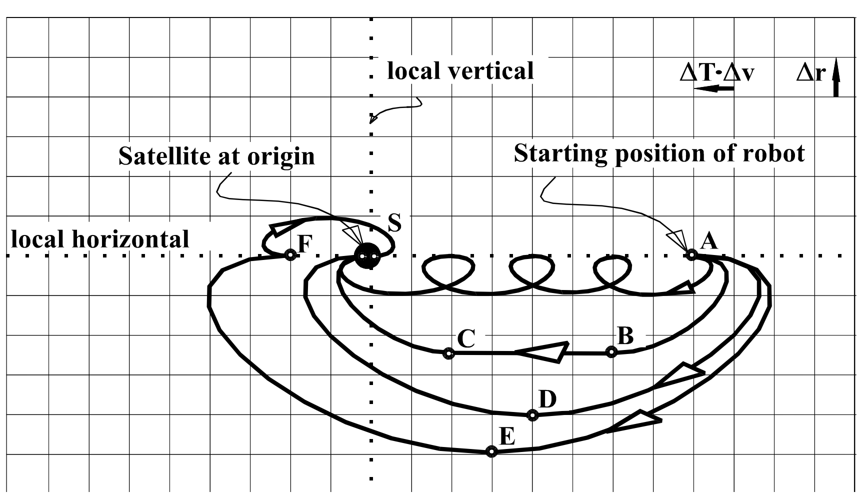
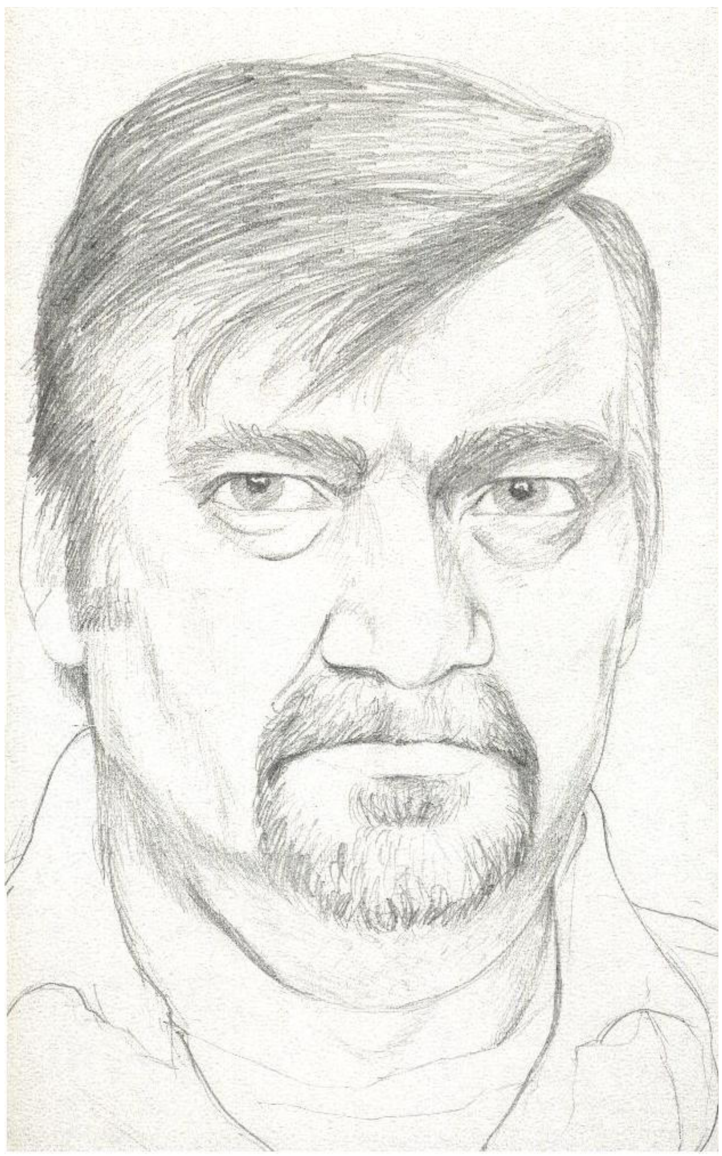


Figure 2: Trajectories in local vertical coordinates of chased satellite

Of course Herb, a PhD candidate from MIT who hired into our group with his nose in the air (my perspective), took offense at my suggestions on the project to which he had been assigned. I suppose he thought that I was just to make positive comments about what a member of our staff had proposed, a loyalty to anyone who happened to be in my group as a part of my review rather than suggesting a simpler way. Maybe.

Herb and I did not really get along well. At one point he made the comment that his receding hairline was proof of the hypothesis that balding occurred because of heightened testosterone levels. I guess he had never heard that it is inherited from one's mother. At any rate, I suggested that in that case he should not run his fingers through his hair so often unless he wanted to be go completely bald.

I think it is fair to say that Herb hated me.



But why would he?