Chapter 25

Matters of Gravity

"When an obscure Russian meteorologist named Alexander Friedmann proposed, in 1922, that the Universe might be expanding, Albert Einstein was sure that he was wrong. Five years earlier Einstein had published a static model of the Universe, and he was still convinced that it was correct. In a rare but dramatic blunder, Einstein bolstered his unfounded beliefs with an erroneous calculation, and fired off a note to the *Zeitschrift fur Physik* claiming that Friedmann's theory violated the conservation of energy. Eight months later, however, after a visit from a colleague of Friedmann's, Einstein admitted his mistake and published a retraction. The equations of general relativity do, he conceded, allow for the possibility of an expanding universe." Guth (1997)

Up until this chapter we have largely ignored what cosmologists have tended to see as the most essential aspects of cosmology – theory, Einstein's hypotheses with regard to space, time, and gravity in particular. These imponderables he integrated at the most basic level of reality. The generalization of his Special Theory of relativity he saw as requiring the incorporation of what had formerly been considered to be but another of the forces of nature between objects within a Euclidean landscape. He removed gravity from its former status as a force transacted *through* space and time like any other, integrating it as an integral feature of the geometrical structure of a spacetime landscape.

We will not dally long in our discussion of complex theoretical considerations including latter day proliferations into string theory and 'multiverses', but we will briefly discuss Einstein's 'cosmological equation. We will also investigate the closure criterion' for the universe as he perceived it, and observe how this has given rise to all manner of conjectures concerning 'missing mass', etc..

We will begin our discussion by venturing into the topic of what Einstein considered to have been his 'greatest error' because it concerns the analogy with classical physics that motivated other aspects of his theoretical considerations. This will shed light on why cosmologists so regularly revive his acknowledged error as a *feature*, rather than a failing, of their own surmisings.

These topics inevitably lead to a discussion of 'dark matter', 'vacuum energy', and why it has seemed reasonable to cosmologists to believe that most of the matter in the universe is not visible. That is to say that they have become convinced that one cannot explain cosmic phenomena without embracing concepts that involve constructs that cannot be directly observed. Some conjectures involve illusive massive particles that, although unaffected by electromagnetic forces, would nonetheless affect and be affected by their surroundings through gravitational effects. Much of this has been by-passed as irrelevant to the work at hand in this volume, although, by providing an alternative resolution to the extreme redshift across galaxy clusters that does not require additional gravitational mass, that is not a problem for the scattering model. To fully resolve this we must also explain why so many cosmologists have been convinced that 'dark matter' must be a reality.

Concepts that preoccupy cosmologists concerning whether dark matter is 'hot' or 'cold' matters primarily to those convinced that there *is*, in fact, dark matter so we will not get into that. Of course, to dispense with such conjectures so off-handedly presupposes resolution by other means. We have concentrated on that alternative resolution, realizing that to a certain extent the intergalactic plasma medium is in itself 'dark matter' about which one is left to conjecture.

Finally we will address that most final of issues, black holes. It is, after all, these vortexes of concentrated matter that are perceived by so many as the penultimate doom of an evolving universe. To propound a stationary state universe with sinks into which matter can be totally removed from consideration would be irrational. We must, therefore, wrestle with these behemoths. So that is the agenda.

a. Einstein's "greatest error"

Einstein's General Theory gets into areas that attempt to explain the universe as a whole. In "cosmological considerations of the general theory of relativity" Einstein (1917) referred to Poisson's well-known equation that applies to gravitation. In particular:

$$\vec{\nabla}^2 \phi(r) = 4 \pi K \rho$$

where the second derivative of the gravitational potential energy $\phi(r)$ is equated to a constant times the mass density ρ as appropriate to inverse square law forces. He noted that there is an apparent incompatibility of this usual formulation and boundary conditions applicable to Newton's theory of gravity. It seemed to Einstein to imply that mass density must approach zero as the extent of the volume to which the equation applies becomes infinite, if the gravitational force were not to become infinite as well. This is certainly mathematically the case.

Clearly, the equation would seem to be incompatible with there being no net force, $\nabla\phi = \vec{0}$ on matter in an extended uniformly dense universe as Newton was wont to accept as reality. This is a view, which Einstein and others have disputed as being erroneous on Newton's part. Further discussion of this situation and how Einstein handled it is provided by Bonn (2008, pp. 130-150). Some of that discussion is duplicated here. It was no doubt to resolve just this quandary that Einstein introduced what he would later acknowledge as having been his greatest error. See Einstein (1952, p. 193), where he states:

> "As I have shown in the previous paper, the general theory of relativity requires that the universe be spatially finite. But this view of the universe necessitated an extension of equations with the introduction of a new universal constant λ, standing in a fixed relation to the total mass of the universe (or, respectively, to the equilibrium density of matter). This is gravely detrimental to the formal beauty of the theory."

Here he audaciously presumes that one's methodologies and theoretical models might appropriately dictate requirements on the *actual* universe that one is attempting to model. This author considers that perspective to be a much more egregious error than what Einstein considered to have been his "greatest" in the above quotation. One must limit their theories and models to valid mathematical descriptions of *actual* phenomena from which to extract invariances and explanations. Theories are not specifications that must be followed by an unwilling universe. Dictums concerning nature must be accepted only to the extent that they *are* valid descriptions if we would have the entire universe acquiesce to such pronouncements. One easily falls prey to gibberish otherwise.

Einstein was concerned because solving Poisson's differential equation for the potential energy of a uniform distribution, resulted in:

$$\phi(r) = 2 \pi K \rho_o r^2$$

which, of course, increases without limit as r becomes very large.

To resolve this problem, he conjectured that there must be some universal constant Λ, defined such that Poisson's equation could be replaced by the following:

$$\vec{\nabla}^2 \phi + \Lambda \phi = 4 \pi K \rho$$

The solution of this equation for a uniform density ρ_o is,

$$\phi_o = - 4 \pi K \rho_o / \Lambda$$

a non-zero constant everywhere. He proceeded to apply a similar kluge to higher dimensionality in his general theory as we will see. Later he would acknowledge this as his "greatest error". It is one that cosmologists continue unabashedly to precisely reincarnate to resolve mismatches between theory and observation.

The author attended a presentation by Philip Mannheim (2008) in which he described, among other topics, the major vagaries of the ill-begotten cosmological constant and how it fits into his own four-dimensional conformal theory of quantum gravity. He stated that not only was inclusion of the lambda term *not* an error, but that it would have been a serious error to have *omitted* it. After his presentation this author asked the presenter privately whether he felt that omitting lambda should be considered Poisson's greatest error instead of Einstein's? He laughed, of course, thought about it for a moment, and then acknowledged quite cheerfully that, yes, he would have to say that. Needless to say, this author does not agree.

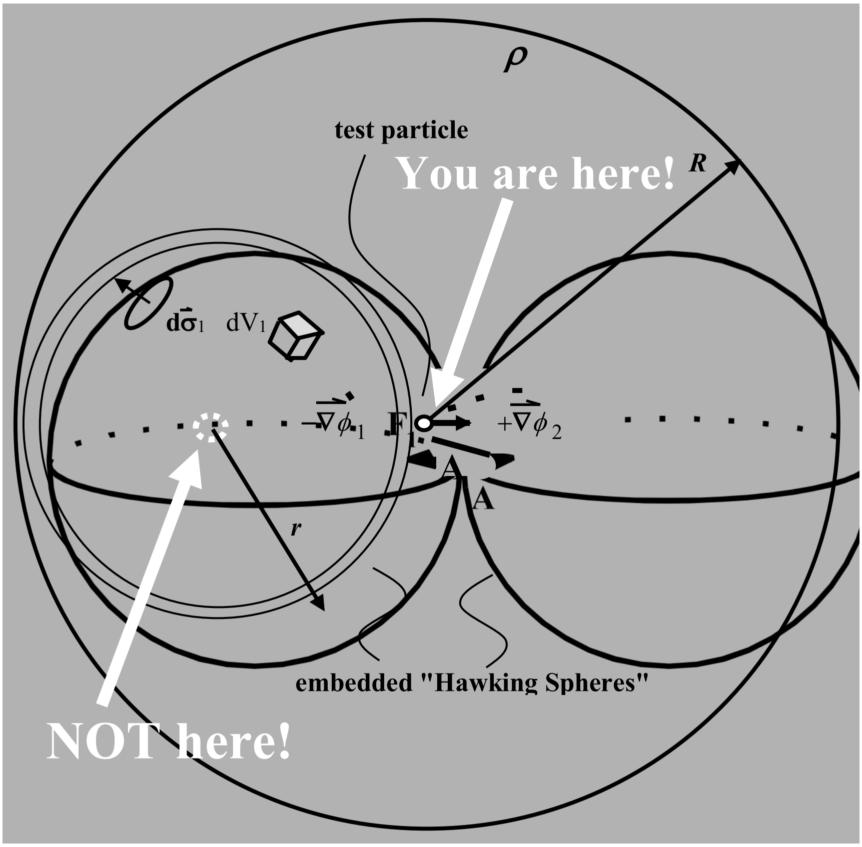

Figure 228: Applying Gauss's integral theorem to embedded 'Hawking spheres'

The situation with lambda is one where we sometimes get so caught up in the mathematical symbolism that we forget to check for an isomorphic physical reality – the association that is the sole justification for any symbolic *representation* at all. Poisson's equation derives from Gauss's integral theorem associated in turn with a divergence theorem discussed in detail in essays by Bonn (2008). This integral theorem illustrated at the left in figure 228 states that:

$$\iint \vec{\nabla}\phi \bullet d\vec{\sigma} = \iiint \rho \, dV$$

Here $\vec{d\sigma}$ is the outwardly directed vector associated with an infinitesimal area on the sphere. The symbol dV represents the infinitesimal volume element within the sphere. The above equation expresses in mathematical terminology that the sum (integral) over an entire closed surface – such as the sphere on the left in figure 228 – of the outwardly-directed perpendicular component of the force field associated with the enclosed mass density distribution is equal to the total amount of mass enclosed by that surface. If the density is uniform throughout the enclosed sphere it corresponds to what Bonn (2008) refers to as a "Hawking sphere". This involves deconstructing the sphere into shells of equal thickness and uniform density to which Hawking (1988, p. 5) referred in siding with Einstein against Newton on the issue of whether an infinite homogeneous universe would necessarily collapse under its own weight. The illustrated shells are artifacts employed in integrating to an infinite limit.

Certainly the perpendicular component of the force field F = $-\vec{\nabla}\phi_1$ due to that portion of the uniform distribution in the left-hand 'Hawking sphere' shown in figure 228 is the same at every point on the sphere. However, in an infinite universe, similar relations apply with regard to F = $-\vec{\nabla}\phi_2$ due to the mass distribution on the right, which is required if we are to maintain symmetry about a test particle on which the field is exerted at point A. Both values $\vec{\nabla}\phi_1$ and $\vec{\nabla}\phi_2$ can be determined using mutually exclusive portions of the mass distribution that maintains the proper symmetry about the test particle by this procedure, and their sum by the rules of field theory is therefore the legitimate solution at point A. So the total force field $-\vec{\nabla}\phi = -\vec{\nabla}\phi_1 + \vec{\nabla}\phi_2$ at the test particle at location A must be zero when we insist on the legitimate application of Poisson's equations to symmetric parts of this problem. And the *proper* way to extend such considerations to the limit of an infinite universe is to let *R* (not just *r*) go to infinity. This gets us away from the troubling necessity of our entire universe being either a gigantic black hole collapsing into a singularity or an equally grotesque, but otherwise required, big bang followed by an expanding universe.

b. the 'cosmological equation'

Einstein was motivated to generalize his work in an attempt to comprehend the universe as a whole, not satisfied with a 'special' theory that dealt exclusively with uniform relative motions that do not characterize much of the reality we observe. His 'world model' of the universe presupposed a finite, static spacetime large enough so that the galaxies, and even clusters of galaxies would constitute insignificant ripples in a uniform mass distribution. To avoid an 'edge' problem to his universe required some alterations. The traditional Euclidean concepts of geometry had to be extended so that three-space could be accommodated as a finite 'surface' within the overall scheme of things. This required a 'metric tensor' to characterize spacetime, into which the concept of gravity itself was incorporated. He developed a tensor differential equation to characterize this model, for which in the limit of small enough volumes, it reverted to something very similar to the usual Poisson equation with Newtonian gravitational force:

$$\vec{\nabla}^2 \phi = 4 \pi \mathrm{K} (\rho + 3p),$$

Dynamic pressure p is included in a stress-energy equivalent. And here again we see the problem that confronted Einstein and that precipitated his error. He needed lambda so his universe would not collapse. Thus, his cosmological equation as cosmologists accept it is:

$$R_{ij} - \tfrac{1}{2} g_{ij} R - \Lambda g_{ij} = 8 \pi \mathrm{G} T_{ij} ,$$

A double subscript indicates the construct is a tensor quantity; R_{ij} and R are functions of the metric tensor g_{ij} and its derivatives, and T_{ij} is the stress-energy tensor that includes the dynamic pressure. We will leave it to others to tell us what this equation implies to them and concern ourselves primarily with whether those implications are realized.

c. the effects of pressure

Certainly gravitational collapse into stars and galaxies occurs. Over (and under) densities will (do) certainly occur for various reasons. In regions of over density gravitational effects will produce contraction into gravitationally bound systems.

But gravity is not the only 'force' to be considered with regard to the resultant behavior of distributions of matter throughout an otherwise uniformly extended universe. There are thermodynamic considerations to be taken into account as well. Any volume of matter at a temperature above absolute zero experiences an outward pressure that would, if it were constrained by a spherical membrane such as in a balloon, for example, force continued outward expansion of that volume in accordance with the following traditional thermodynamic formula:

$$p\,V = n\,\mathrm{k}\,T$$

where p is thermodynamic pressure, V is the volume within the surface, n is the number of particles of gas within the volume, k is Boltzmann's constant, and T is the temperature of the gas within the volume. This produces a force on each unit of the surface area as shown in figure 229; the associated force f_p would be experienced in an outward direction due to the thermodynamic pressure, p, where r_J indicates the radius of the enclosed overdensity volume.

This force would be countered in such a hypothetical situation by the gravitational force operative on the matter contained within the volume as assumed by Einstein's analysis of an inward gravitational force. This would reduce the volume, raise the kinetic temperature, thereby increasing outward pressure.

The *Jeans criterion* for collapse takes both forces into account in assessing the condi-

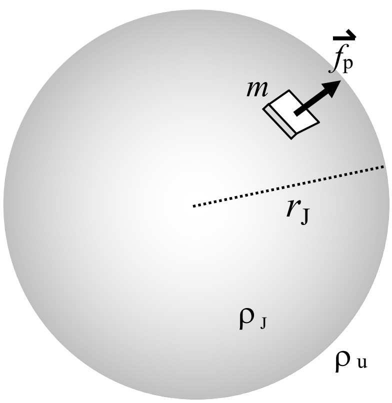

Figure 229: Thermodynamic forces active in the structure- producing processes of the universe

tions throughout the volume for overall stability. The resulting criterion for over density in an ideal gas with no external forces, is

$$r_J > \sqrt{\gamma\, k\, T / m_p\, G\, \rho_J}$$

The symbols k, T, and G are as defined previously. Here r_J is the *Jeans length* beyond which gravitational collapse would be inevitable, γ is the adiabatic expansion factor (approximately unity), m_p is average molecular mass, and ρ_J is the mass over density.

$$\rho_J \equiv \rho_M - \rho_u$$

which is merely the amount of density in excess of the immediate surroundings.

Thus, we could define regions of interplay between the forces of gravity and thermodynamic considerations.

$$r_J \cong 1.5 \times 10^8 \sqrt{T / \rho_J},$$

At a minimum, it must be obvious to the reader that there are various forces that have acted in concert in determining how our universe has come to its observed current conditions. One might theorize various models to derive conditions we perceive as essential to the universe we observe, but such models are of little significance relative to factual observation.

What is observed is a pattern of structures for which any conceivable aggregation process from a uniform distribution of galaxies would take upwards of ten times longer than the alleged age of the universe according to any version of the standard model.

d. uniformity of matter in the universe

In any case, Einstein was persuaded that the universe must indeed be homogeneous and very uniform at distances large with respect to our own galaxy and its immediate environs such that a uniform density seemed a reasonable assumption. That assumption seems even more valid now that many billions of galaxies have already been observed and mapped. See figure 230 from Maddox et al. (1990) and refer again to diagrams provided as figures 5 through 7 in chapter 2 above. The two-dimensional map of the sky provided in figure 230 covers a region 100° by 50° around the South Galactic Pole. Automatic Plate Measuring (APM3) that provided this galaxy survey provided the positions, magnitudes, sizes, and shapes for about 3 million galaxies. Each pixel covers a small patch of the sky that is 0.1° on a side. The image is shaded according to the number of galaxies per pixel area. The pixels are brightest where there are the most galaxies. Clusters, containing hundreds of galaxies are seen as merely a bright patch. Larger elongated bright patches are 'superclusters' and 'filaments'. Small empty 'holes' are excluded viewing regions around bright foreground stars in our own galaxy, nearby dwarf galaxies, and globular clusters. Clearly the galaxy distribution by angle seems quite uniform on the sky.

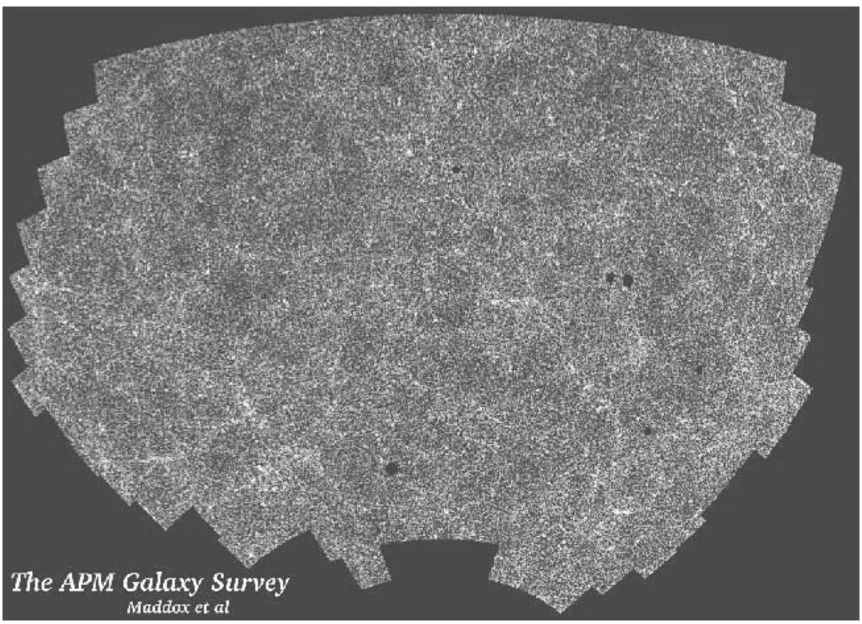

Figure 230: The manifest uniformity of the universe at large enough scales

But Einstein was also convinced by his interpretation of Poisson's equation that the universe must be finite to keep the velocities of distant galaxies within bounds. That is a conviction we must question – not for reasons of Hubble's hypothesis, which ultimately persuaded him to disavow his arbitrary insertion of Λ, but for physical and mathematical reasons we have just discussed.

e. expanding universe hypothesis

Hubble's hypothesis did seem to have come to Einstein's rescue with regard to the universal constant Λ such that, assuming an extreme initial velocity of the matter at remote distances, one could suppose that gravitation was indeed operative at these extreme ranges in bringing velocities of distant galaxies into check.

That would seem to put us at a central non-Copernican position in spacetime. However, in the four-dimensional geometrical approach of the general theory, our place in three-space would still be equivalent to any other. Our place in time is quite another matter. We would occupy a very unique place in the history of the universe as Hawking (1988) noted by the title of his popular best seller, "A brief history of time – from the big bang to black holes".

At any rate it was Hubble's hypothesis of expansion of the universe that effectively did away with any need for Λ in Einstein's mind, especially in a finite universe. So he acknowledged that it had all been a bad mistake – that he should have let his equations guide him without fear that the universe might not follow. He recognized that Hubble's constant provided a means for assessing gravitational values of cosmological significance including the average density and radius of the entire universe. His cosmology was conceived somewhat as shown in cartoon form in figure 231.

Clearly Einstein's "greatest error" was purosely incorporated to avoid gravitational collapse. It retains this role in models that have resurrected it. Inflation and the recent discovery that at great distances 'expansion rate' seems actually to be increasing rather than decelerating has emboldened its reincarnation and, in some applications, turned it into a variable rather than a constant. See for example, Bothun (1998) who says, "In the cosmological equations Λ appears as a long-range repulsive term and acts like a source of negative pressure," thus adding another 10 – 20 % to the presumed age of the universe.

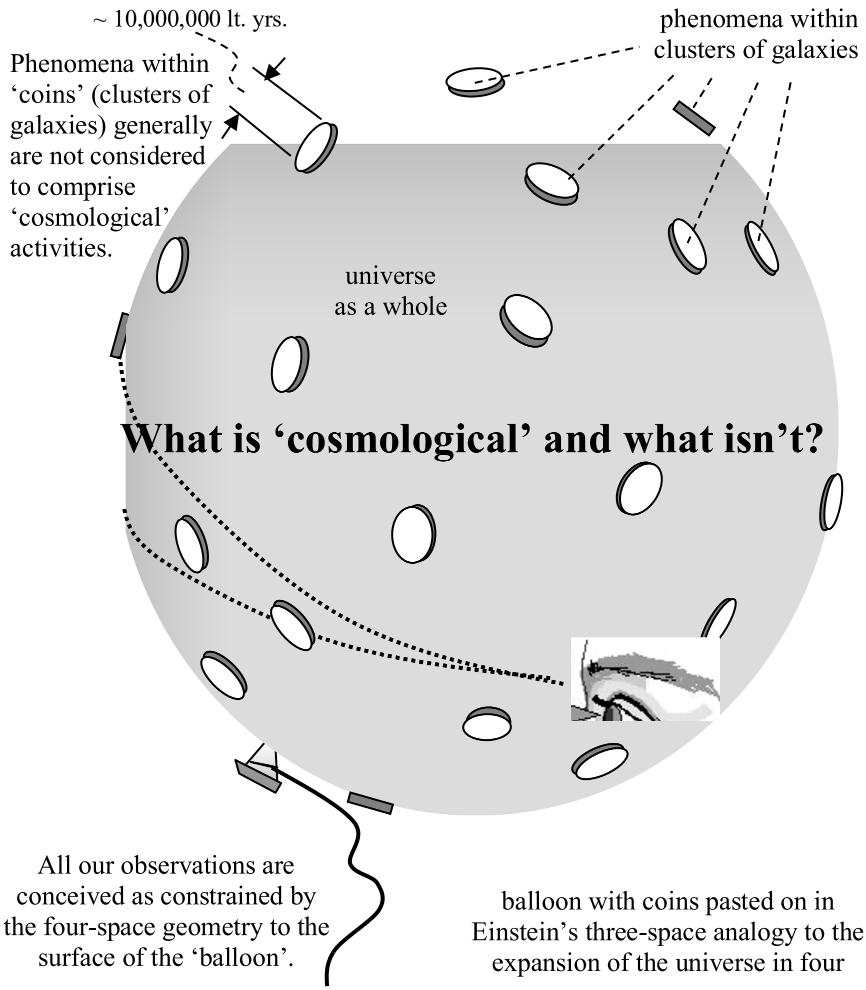

Figure 231: Visualization of Einstein's conception of a four-space universe

f. the 'critical density'

Although the *critical density* and its derivation are cornerstones of what general relativity and current cosmology are all about, it is a simple concept with a correspondingly simplistic, non-relativistic, derivation of its value. The derivation begins with the classical concept of 'escape velocity' from a massive body such as earth and proceeds to considerations of distant objects receding at extreme velocities as a part of the expanding universe hypothesis. The collective mass of the universe is hypothesized as the retro force keeping expansion from getting out of hand. Of course this derivation does not get into rationale for the strange initial condition, its cause, nor yet the criticality of the tuning of the model that is required just to realize this condition. That was addressed by Guth.

In classical physics the kinetic energy, T, of an object of mass m that is moving with velocity v is,

$$T = \tfrac{1}{2}\, m\, v^2$$

The gravitational potential energy, V, of an object of mass m at a distance, r, from the center of gravity of a spherical mass distribution of total mass M is,

$$V = -\, G\, M\, m\, /\, r.$$

Here G is Newton's gravitational constant we defined earlier.

An object will escape the gravitational field of the distributed mass if its kinetic energy exceeds the absolute value of the gravitational potential energy by which it is bound, such that:

$$\tfrac{1}{2}\, m\, v^2 \geq G\, M\, m\, /\, r.$$

Kinetic energy will be converted into gravitational potential energy as it is slowed down in proceeding further from the center, satisfying the energy conservation law. If the two forms of energy happen to be equal then the object would come to a stop at a very great distance with essentially zero velocity and zero potential energy.

It is virtually the same calculation for stars and dust circulating about a galaxy or galaxies in a cluster whose escape would be from the attraction of all the other galaxies and intragalactic gases. And it is the same equations that would be applied to a finite (Hawking sphere) universe that is in question. Will they stop, turn around, or fall back to swirl with the other galaxies until finally they dissipate their energies and collapse into a gigantic black hole?

If there is just enough material in the universe to stop the galaxies, then perhaps the universe will go on forever expanding ever more slowly – never escaping and never collapsing. Einstein preferred that solution for obvious reasons. And the universe seems to have acquiesced amazingly well, although apparently not quite. How it could be so close – and yet so far – is one of the difficulties facing standard model cosmologists

According to Hubble's law the approximate velocity of a distant galaxy is proportional to its distance, $v = c\, H_o\, r$, so the kinetic energy of a galaxy can be written $\frac{1}{2}\, m\, (H_o\, r)^2$. The mass of all the material inside a sphere of radius r is given by $M = 4/3\, \pi\, r^3\, \rho$, where ρ is the average density of the universe. Substituting these two expressions into the inequality provided above produces the inequality:

$$\tfrac{1}{2}\, m\, (H_o\, r)^2 \geq G\, (4/3\, \pi\, r^3\, \rho)\, m / r.$$

By simplifying and rearranging to solve for the critical situation for which equality applies with $\rho = \rho_o$, we obtain:

$$\rho_o = 3\, c^2\, H_o^2 / 8\, \pi\, G$$

This 'critical density' depends only upon universal constants. It is approximately 8×10^{-30} gm cm^{-3}.

Interestingly, from the usual standard model understanding that the radius of the universe is equal to the Hubble distance, $r_u = 1 / H_o$, we can determine the 'critical' Schwarzschild radius r_s of the universe itself, since $v \rightarrow c$ as $r_u \rightarrow r_s$, as follows:

$$r_s = 2\, G\, M_u / c^2 = 2\, G\, (4/3\, \pi\, r_u^3\, \rho_u) / c^2$$

$$= 8\, \pi\, G\, \rho_u / 3\, c^2\, H_o^3 = 1.74 \times 10^{56}\, \rho_u$$

So that, if $\rho_u = \rho_o$, then the universe would be neatly tucked into its own black hole. But, of course, $\rho_u = \rho_o$ is by no means confirmed, and in fact $\rho_u < \rho_o$ seems to be the case. Since according to the standard cosmological model the same amount of mass has existed in smaller and smaller confines in the past, this means that the entire universe would have to have emerged from the confines of a gigantic black hole in the very recent past by cosmological standards.

g. the missing matter

Estimates of the mass of stars and galaxies comprised of them have been obtained using methods described previously in chapter 16. By adding the masses of all clusters and individual galaxies in observed regions and dividing by the volume of space involved in the survey one obtains an estimate of ρ_u. As larger and larger regions of space are included in such surveys the mean density of baryonic mass has approached a figure more like 5 x 10^{-31} than a value significantly greater than 10^{-30} gm cm^{-3} as Einstein would have preferred. Certainly there is a fairly large degree of uncertainty or 'wiggle room' in this value because it is based on a series of estimations that do not do too well on accounting for dispersed plasma, but the degree to which there is a shortfall is quite appreciable. This gives rise to many heated discussions of 'missing mass' that inevitably devolve into discussions of 'dark matter' and the even more mysterious 'vacuum energy'.

By any accounting the observations imply an 'actual' density of the universe that is a relatively small fraction of Einstein's 'critical density'. This in turn should imply that the universe will not collapse back onto itself according to those same theoretical considerations. There are discrepancies in behavior from what is predicted by standard models that have promoted the various 'dark matter' theories, of course, which some believe ups that percentage a little closer to 100 percent. At any rate Einstein's 'greatest error' continues its ill-begotten success, suggesting to those who should know better that a mysterious 'vacuum energy' might save the day.

If the critical mass density is not realized – as it evidently is not, other than in mysterious ways – we have succeeded in escaping from the biggest of all possible black holes, a supposedly impossible feat.

So what's to doubt?

h. inherent problems in the theory

Let's just list a few of the objections to the certitude given this bit of cosmic mysticism that constitutes the theoretical underpinnings of the standard cosmological model. These objections are not necessarily listed in the order of the significance the author places on them:

1) There are the inconsistency problems in observed data – stars in our own galaxy that are older than, or nearly as old as, the supposed age of the universe, too early giant elliptical galaxies, and other too early structures throughout the universe, which are continuously being excused away as even earlier representatives are found.

2) Supposed requirements for 'dark matter' to support the virial theorem calculations with regard to galaxy and galactic cluster dynamics.

3) Evidence for acceleration (rather than deceleration) of expansion on which the whole calculation is based – revitalizing Einstein's 'mistake'.

4) A general willingness to entertain Einstein's admitted "greatest error" or any other alteration of time-honored principles, laws of physics, or universal constants just to make calculations work.

5) Theoretical inconsistency with black hole theory since the universe by these calculations has been a black hole for most of its existence but is now apparently emerging from that ultimate lethality, contradicting notions put forward by the same theorists in the context of black holes being inescapable.

6) The current understanding that gravitational forces are transmitted via gravitons in analogy to photons transmitting electromagnetic forces, suggests that these must also be limited to speed-of-light travel and involve wavelengths and frequencies proportional to the momentum and energy transmitted. This would certainly be associated with redshifting in accordance with Hubble's hypothesis with an associated diminution of both with distance if the effect is geometry-dependent. So that to presume unabated inverse square gravitational forces or its equivalent in the general theory (like the otherwise inverse square luminosity relationship) at such extreme distances seems unwarranted in the case of gravitation also.

7) The critical density calculation is based on an arcane model of the universe as discussed above with regard to an inappropriate application of the divergence theorem to all space and Poisson's law to the universe as a whole.

The first six of these are more or less nitpicking. The seventh addresses underlying assumptions of the theory with respect to incorrectly applying gravitation to cosmology, and virtually all current theoretical thinking in cosmology. There is no reason to believe the underlying assumption should be considered valid for the universe as a whole. Presumed validity in this domain is based on precedence in other domains and the reputations of those who have previously made the assumption, perhaps most notably Einstein and Hawking.

[*All references are included as a bibliography of the parent document. I include only:*

Bonn, R. F., The Aberrations of Relativity, ISBN 978-0-6151-9781-4, Vaughan Publishing, Seattle (2008)

This is a book by the current author of articles and essays concerning relativity.]