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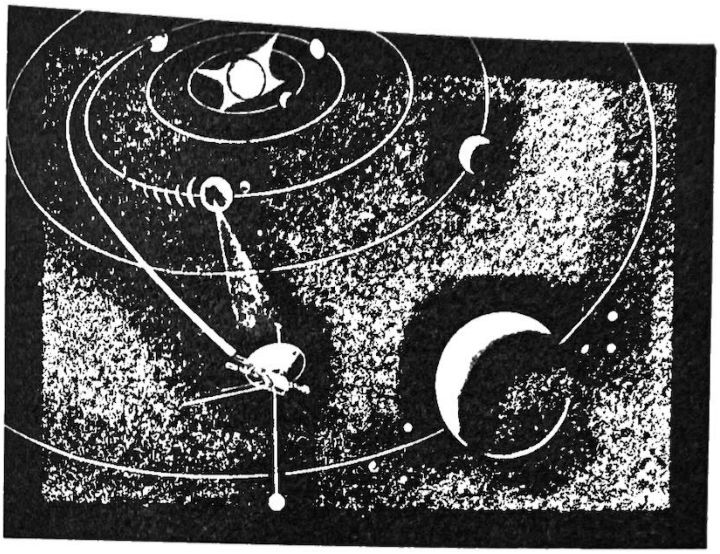

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SKEWED VS ORTHOGONAL REACTION WHEELS FOR OUTER PLANET EXPLORATION

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SKEWED VERSUS ORTHOGONAL REACTION WHEELS FOR OUTER PLANET EXPLORATION *

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Reliability analysis shows that skewed redundant reaction wheels have a significantly greater potential for long life than orthogonally mounted redundant reaction wheels for outer planet exploration (OPE) spacecraft attitude control. Potential mechanization problems associated with the implementation of skewed reaction wheels are considered. The topics discussed include skewed reaction wheel sizing, redundancy switching, control laws, desaturation, and failure detection. Control and fault isolation equations, and redundancy switching logics are presented for skewed and orthogonal systems. On-board computer software is defined for solution of the control law, fault isolation and wheel switching software, and a comparison between skewed and orthogonal attitude control computer requirements is made.

The mechanization difficulties associated with the skewed configuration are minimal, because coordinate transformation and redundancy switching are easily accomplished within the on-board digital computer. It is shown that the impact of the computational differences on computer reliability is small. It is concluded that the skewed redundant reaction wheel configuration has greater potential for the long life OPE missions than orthogonal configurations.

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INTRODUCTION

Outer Planet Exploration spacecraft attitude control system design objectives must be focused on achieving long life, endurance, and operability. Reaction wheels are considered a prime candidate for OPE attitude control[1] because they can provide a planet scanning capability for strapdown experiment packages by maneuvering the spacecraft to obtain the desired line-of-sight to the planet. This eliminates the need for potentially unreliable platform gimbals and servos. Redundant reaction wheels are required, because OPE mission times are long compared to the actuator MTBF.

The standard approach for redundant reaction wheels[2] is to mount an extra wheel on each axis of an orthogonal triad. Two successive wheel failures could result in loss of control. Recent investigations of skewed redundant sensor configurations[3,4] motivated consideration of a skewed redundant reaction wheel configuration. For such a set (of six wheels) any three wheels could fail without loss of control.

Many skewed sensor configurations have been investigated with the objective of choosing a skewed configuration to minimize measurement errors. It has been shown[5] that the optimum configuration is dependent upon the desired accuracy envelope. In an analogous fashion, the skewed reaction wheel configuration is dictated by the momentum storage envelope associated with the mission. It is shown in reference (5) that accuracy is degraded for adjacent skewed sensors. Similarly, momentum absorption and torque output capability are reduced for adjacent skewed actuators.

In this paper several orthogonal and skewed reaction wheel configurations are postulated which meet the OPE momentum storage envelope. An objective of the paper is to present a detailed assessment of the problem of momentum storage and torque output reduction associated with skewed actuators. Reliability comparisons will be made between equal-performance orthogonal and skewed reaction wheel systems.

The results of the reliability analysis indicate that skewed configurations are preferred for OPE. Before this conclusion is finalized, however, consideration must be given to several possible implementation problems associated with skewed reaction wheels.

Wheel torque commands formulated from error signals in a body fixed orthogonal coordinate frame are directly related to reaction wheels mounted in the body coordinates. Parallel operation of redundant orthogonal wheels is accomplished by simple allocation of commands between two or more colinear wheels.

Transformation of torque commands into skewed wheel coordinates varies for each combination of skewed actuators. The transformation is not unique for parallel operated redundant skewed actuators. A pseudo-inverse skewed wheel control law is presented in the paper, which minimizes computer storage associated with coordinate transformation, and allows parallel operation of skewed wheels.

For high maneuver rate missions, wheel gyroscopic cross coupling results in increased power utilization and degraded experiment pointing accuracy. The decoupled control law presented by Cannon [6] can be utilized to minimize the effects of cross coupling. An extension to this control law is suggested for decoupled control of a skewed redundant reaction wheel control system.

Failure detection for actuators operated in parallel is less straightforward for skewed than for orthogonal wheel configurations. Gilmore [3] discussed a parity test logic approach to failure isolation of redundant sensors. The parity test scheme can be applied to the failure diagnosis of skewed reaction wheel. However, an alternate scheme which makes use of a property of the pseudo-inverse control law is recommended.

To evaluate the reliability impact of the computational differences between the skewed and orthogonal configurations, computer programs were designed and coded in a typical aerospace computer assembly language. Data are presented which indicate the implied differences in computer storage and cycle time requirements. From these data an assessment is made of the impact on computer reliability by the skewed reaction wheel implementation.

RELIABILITY COMPARISON

Wheel Sizing

Outer Planet Exploration vehicles of the 2000 lb class are typically disc shaped devices. Maneuver rates for scanning have been identified on the order of 1°/sec as a possible maximum. The 1°/sec requirement results in a necessary momentum storage capability of 7.5 ft lb sec about any axes in the YZ plane, and 1.5 ft lb sec about the imaging axis. The 1.5 ft lb sec requirement about the X axis is the result of accumulating the maximum cross coupling torque, due to having 7.5 ft lb sec stored in the Y axis, while maneuvering about the Z axis at 1°/sec for 10 seconds.

For mission times which are long compared to proven reaction wheel MTBF, implementation of redundant wheel schemes will improve the overall attitude control system reliability. The two configurations with suitable momentum storage capability initially considered are: a redundant orthogonal configuration, which provides duplication of a standard three wheel orthogonal set, and; a six wheel skewed configuration with the spin axes defining a cone and lying 60 degrees apart in plan view as shown in Fig. 1.

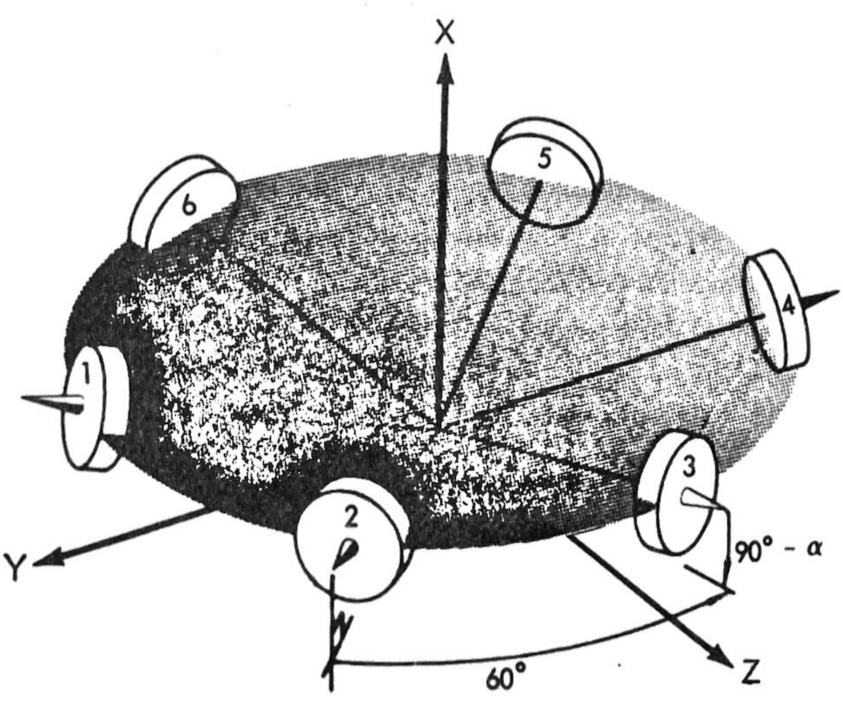

Fig. 1 SIX WHEEL CONICAL REACTION WHEEL CONFIGURATION

Although both systems could be operated in a parallel redundant mode, only standby redundancy is considered due to the higher inherent reliability. The conical skewed configuration will be shown to be more reliable than the doubly redundant orthogonal configuration because of its capability to withstand any three failures, however, there are significant sizing implications against the skewed configuration which must be examined. The wheel to body axes momentum transformation for the six wheel skewed configuration is defined as

$$\vec{h}_B = \begin{bmatrix} C\alpha & C\alpha & C\alpha & C\alpha & C\alpha & C\alpha \\ S\alpha & S\alpha/2 & -S\alpha/2 & -S\alpha & -S\alpha/2 & S\alpha/2 \\ 0 & \sqrt{3}S\alpha/2 & \sqrt{3}S\alpha/2 & 0 & -\sqrt{3}S\alpha/2 & -\sqrt{3}S\alpha/2 \end{bmatrix} \vec{h}_i$$

$$B = x, y, z \qquad i = 1, 2, \cdots, 6$$

where $S\alpha$, $C\alpha$ are the sine and cosine functions of the cone half angle respectively.

There are twenty unique combinations of the six wheels taken three at a time without respect to the ordering in the permutation. Therefore, there are twenty 3 x 3 matrix transformations to be examined to determine the skewed wheel momentum requirements in terms of vehicle momentum requirements in body axes. For example, if skewed wheels 1, 2, 3 are used for spacecraft control, the wheel momentum requirements are defined as

$$\vec{h}_{123} = Q_{123}^{-1} \vec{h}_B$$

where

$$Q_{123} = \begin{bmatrix} C\alpha & C\alpha & C\alpha \\ S\alpha & S\alpha/2 & -S\alpha/2 \\ 0 & \sqrt{3}S\alpha/2 & \sqrt{3}S\alpha/2 \end{bmatrix} \quad \text{and} \quad Q_{123}^{-1} = \begin{bmatrix} 1/C\alpha & 0 & -2/\sqrt{3}S\alpha \\ -1/C\alpha & 1/S\alpha & \sqrt{3}/S\alpha \\ 1/C\alpha & -1/S\alpha & -1/\sqrt{3}S\alpha \end{bmatrix}$$

The rows of the twenty Q^{-1} matrices must be examined to determine the largest momentum gain factor for each skewed wheel. There are twelve types of rows which appear in the Q^{-1} matrices as summarized in Table 1.

By inspection, the rows resulting in the maximum multiples are rows 2 and 7. These rows appear only in Q^{-1} matrices associated with adjacent wheels.

Table 1

TYPES OF MOMENTUM REQUIREMENT TRANSFORMATION ROWS

1	$1/C\alpha$	0	$2/\sqrt{3}S\alpha$	5	0	0	$2/\sqrt{3}S\alpha$	9	$1/3C\alpha$	$1/3S\alpha$	$1/\sqrt{3}S\alpha$
2	$1/C\alpha$	$1/S\alpha$	$\sqrt{3}/S\alpha$	6	$1/2C\alpha$	$1/2S\alpha$	$1/2\sqrt{3}S\alpha$	10	0	$1/S\alpha$	$1/\sqrt{3}S\alpha$
3	$1/C\alpha$	$1/S\alpha$	$1/\sqrt{3}S\alpha$	7	$1/C\alpha$	$2/S\alpha$	0	11	$1/2C\alpha$	$1/S\alpha$	0
4	$1/2C\alpha$	$1/2S\alpha$	$\sqrt{3}/2S\alpha$	8	$1/3C\alpha$	$2/3S\alpha$	0	12	$1/2C\alpha$	0	$1/\sqrt{3}S\alpha$

The momentum absorption envelope is based upon the requirement to maneuver about any axis in the YZ plane at 1°/sec. The momentum multipliers of rows 2 and 7 require the skewed system to have four wheels each with 1.732 times the momentum storage capacity, and two wheels each with 2 times the momentum storage capacity of the comparable orthogonal wheel. For typical reaction wheels of the size under consideration,

$$\text{Weight} = KH^{1/2}$$

Hence, the skewed system would suffer a 1.35 weight factor penalty over a six wheel orthogonal system. The slightly heavier nine wheel orthogonal configuration would have a superior reliability.

Assuming that wheels are not sized for meeting the momentum absorption requirement with adjacent wheels, the momentum requirement transformation with the next largest momentum multiplier is row 5 with a momentum gain of 1.2 and a resulting weight factor of 1.1.

In considering the reliability effects of minimizing the use of adjacent wheel sets it is necessary to postulate a wheel usage criterion that is relevant to the OPE mission, and to then develop a compatible wheel switching philosophy.

The wheel sizing criteria depend on the vehicle maneuverability requirement during near encounter sequences. These periods constitute a very minor fraction of the total mission time. During the long cruise periods wheel size requirements are insignificant. Consequently, the postulated three wheel utilization will allow adjacent wheel operation during cruise, but will consider the necessity of operation with an adjacent three wheel set during a near encounter sequence to be termed a failure. With these constraints, a failure mode switching criterion has been developed and the reliability impact assessed.

Failure Mode Switching

A simplified failure mode switching approach for the six wheel conical configuration is shown in Fig. 2.

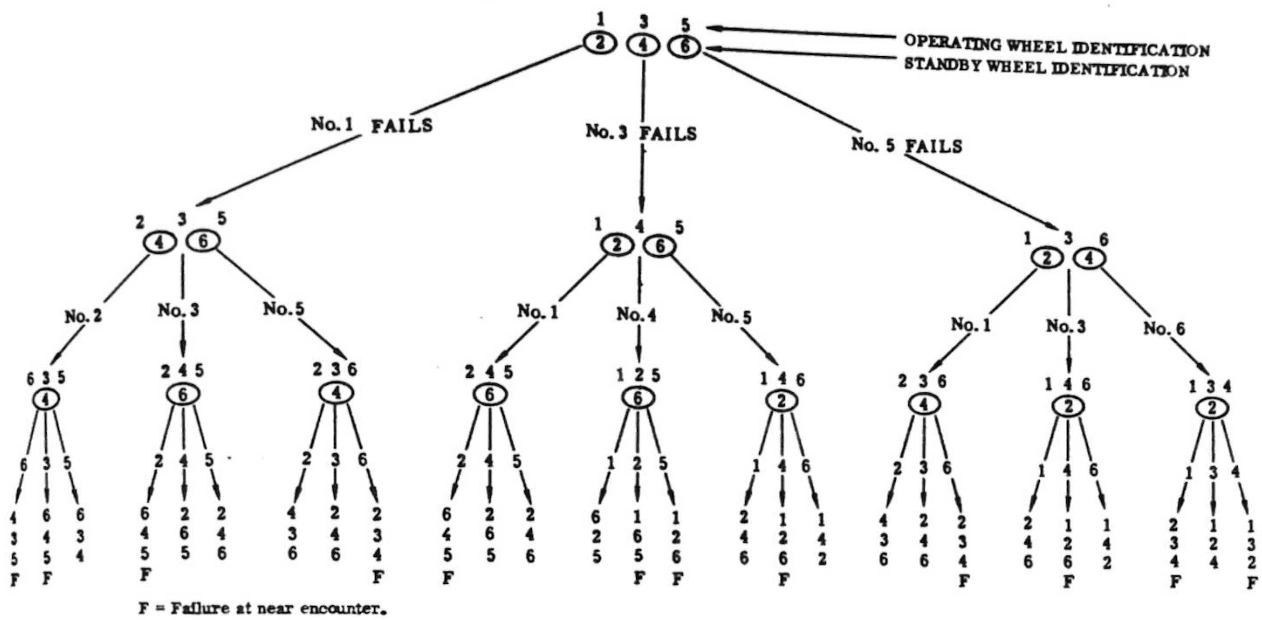

Fig. 2 FAILURE MODE SWITCHING LOGIC WHICH FORESTALLS ADJACENT WHEEL FAILURE MODES TO THE 3rd WHEEL FAILURE

The initial operating mode involves a configuration of three wheels with maximum spacing and a dedicated backup for each operating wheel. After a single wheel failure, only the dedicated backup for the failed wheel is switched into operation. At the failure of a second wheel, the backup wheel, situated closest to that which has failed, is switched into operation. At the occurrence of the third wheel failure one of twenty-seven equally likely operating modes will result. Fifteen of these operating modes do not involve three adjacent wheels, and therefore would still provide system operability. This failure mode switching approach is not optimum for the six wheel skewed configuration.

As pointed out above, the OPE wheel sizing was based on vehicle maneuverability requirements at near encounter. It was assumed, for sizing purposes, that adjacent reaction wheels would represent a system failure at near encounter. Any combination of three wheels could be used, however, for the long cruise periods where maneuverability requirements are low. Fig. 3 defines a failure mode switching philosophy, which takes advantage of using adjacent wheel combinations for one and two wheel failure modes to minimize the number of adjacent wheel operating modes after three wheel failures.

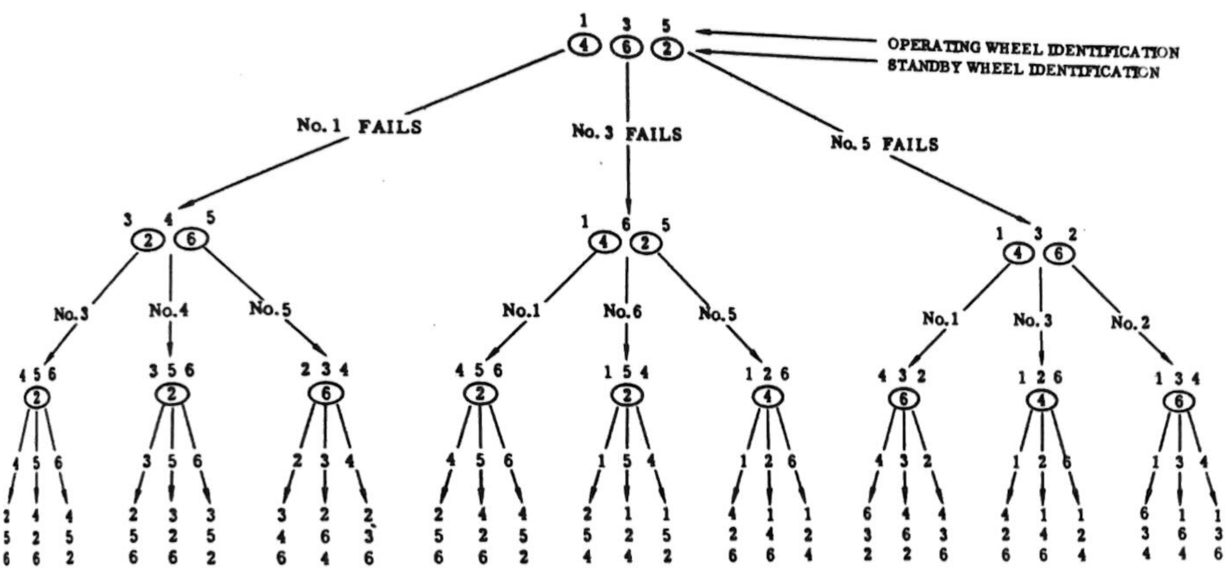

Fig. 3 DEFENSIVE DEGRADED FAILURE MODE SWITCHING LOGIC WHICH RESULTS IN NO ADJACENT WHEEL FAILURE MODES AT THE 3rd WHEEL FAILURE.

The system was initialized in a maximum wheel spacing configuration. The dedicated backups to this initial configuration were selected such that one of the single wheel failure modes resulted in adjacent reaction wheels. At the second wheel failure, the system was configured such that the resulting operable wheels were adjacent wherever possible. If a near encounter occurred before a three wheel failure with the system in an adjacent wheel configuration, the wheels could be switched to achieve a non-adjacent configuration. Therefore, only three wheel failure modes need be considered in determining the reliability impact. This redundant wheel switching philosophy shown in Fig. 3 results in no three wheel failure modes consisting of adjacent wheels.

Reliability Enhancement Evaluation

A system which can tolerate any combination of up to n component failures has a reliability

$$R(t) = \sum_{i=0}^{n} P_i$$

where P_i is the probability of exactly i component failures occurring by time t. In general the reliability of a system which can tolerate only certain combinations of up to n component failures is

$$R(t) = \sum_{i=0}^{n} k_i P_i$$

where k_i is the probability of an i-failure situation being a viable combination of component failures.

For the success criterion which tolerates any combination of up to three failures except those which result in only three adjacent wheels being operable, $k_i = 1$, for $i < 3$, and k_3 depends on the failure mode switching philosophy applied. For the simplified and optimum failure mode switching implementations illustrated above for the six skewed wheel system, k_3 equals 15/27 and 1 respectively. It is shown in reference (7) that for this case:

$$P_i = \frac{(3\lambda t)^i}{i!} e^{-3\lambda t}, \text{ where } i = 0, 1, 2, 3$$

The resulting reliability curves are shown in Fig. 4.

It should be noted that the six wheel skewed system sized such that having only three adjacent wheels operational at a near encounter represents a mission failure is as reliable as the system with no permutation limitations, if an orderly switching scheme is used for degraded operating modes. Therefore, it can be said that a six wheel skewed scheme can be implemented which will have greater reliability, more than equal performance, and only 10% increase in weight over the six wheel orthogonal system.

In considering the application of the six wheel reliability data to the real world of currently available reaction wheels, it is obvious that a highly reliable OPE attitude control system will require more than six wheels. Although the Orbiting Geophysical Observatory (OGO) reaction wheels have survived continuous ground testing in excess of eight years, it is hard to believe that reaction wheels will have

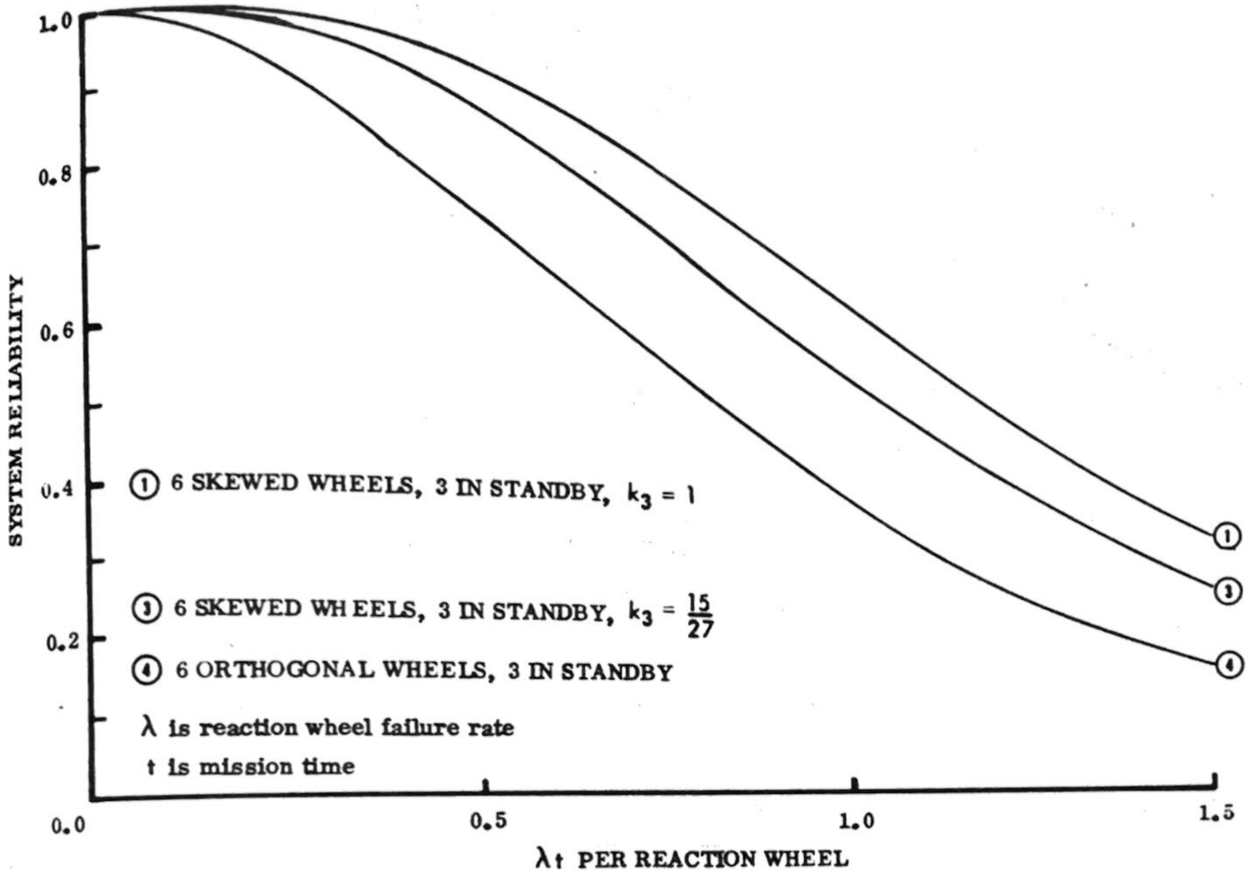

FIG. 4 SYSTEM RELIABILITY AS A FUNCTION OF λt FOR 6 WHEEL ORTHOGONAL AND SKEWED CONFIGURATIONS.

failure rates so low that a six wheel system can achieve a .9 reliability for a twelve year mission. Consequently, the above analysis was extended to consider nine orthogonal wheels, and seven and eight skewed wheel systems. The comparable reliability versus λt plots for these cases are shown in Fig. 5. The skewed configurations are given for three wheel operability. Assuming an intelligent switching philosophy, however, situations resulting in only adjacent operational wheels could be considered system failures for the clearly superior eight wheel system, with no impact on reliability as shown for the six wheel skewed system.

The weight factor associated with the eight wheel skewed system allowing adjacent wheels is obtained from the appropriate Q^{-1} matrix, with

$$h_{max} = \frac{H_x}{C\alpha(2-\sqrt{2})} + \frac{H_y}{S\alpha(2-\sqrt{2})} + \frac{H_z(1+\sqrt{2})}{S\alpha(2-\sqrt{2})}$$

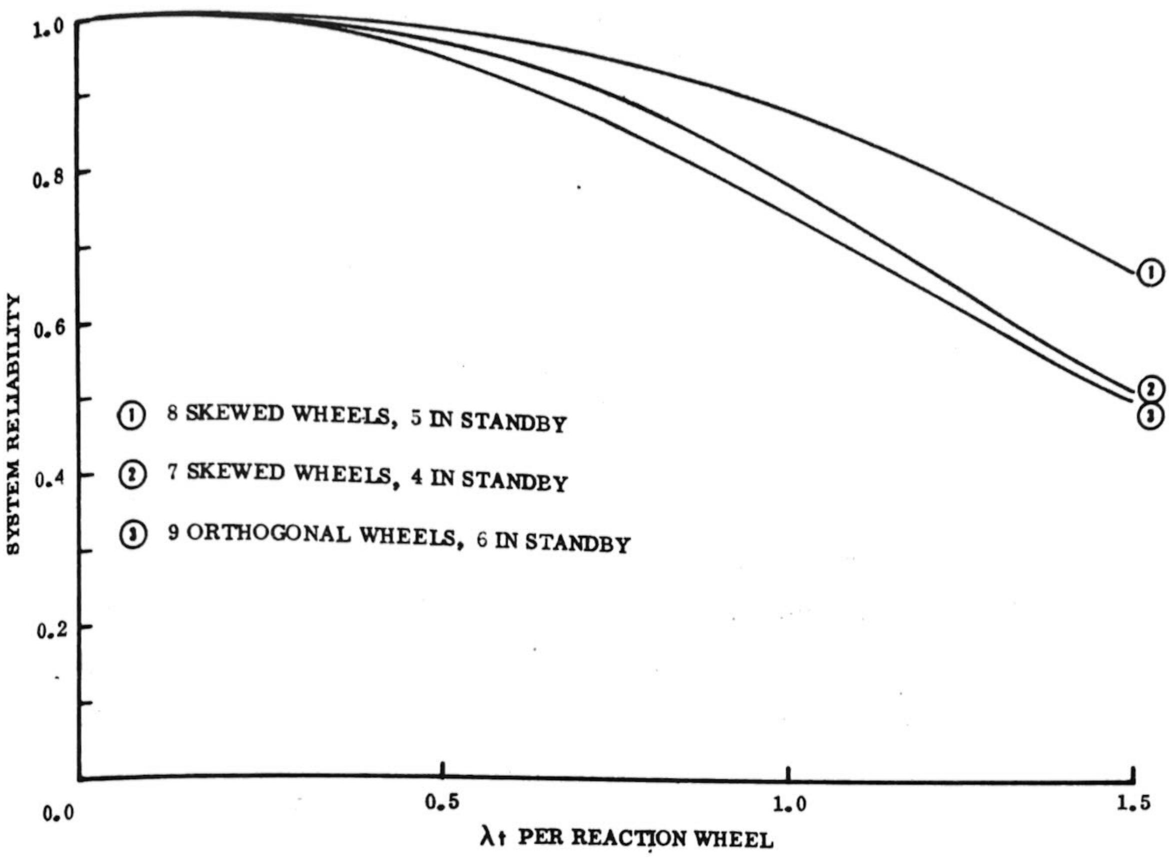

FIG. 5 SYSTEM RELIABILITY AS A FUNCTION OF λt FOR EIGHT AND SEVEN WHEEL SKEWED AND NINE WHEEL ORTHOGONAL CONFIGURATIONS.

The momentum absorption envelope is pill shaped with radius of 7.5 ft lb sec. The maximum momentum gain factor is associated with a maneuver about the Z body axis. For half cone angle, α = 75 deg, the momentum gain factor is

$$h_{max} = 4.2H$$

such that the weight factor is 2.05. For the next worse case configuration, i.e.; two adjacent wheels, with the third displaced 90 deg from one of the two adjacent,

$$h_{max} = \frac{h_x}{\sqrt{2}C\alpha} + \frac{h_y}{\sqrt{2}S\alpha} + \frac{h_z(1+\sqrt{2})}{\sqrt{2}S\alpha}$$

For a maximum rate maneuver about the Z axis,

$$h_{max} = 1.7\,H$$

The weight factor is 1.3. The eight wheel skewed system, therefore, weighs 18% more than the nine wheel orthogonal configuration, but has a far better reliability expectation. It is expected that a redundancy failure switching scheme could be developed which would allow a further reduction of the eight wheel skewed configuration weight without a significant reduction in reliability.

IMPLEMENTATION CONSIDERATIONS

Although there are reliability advantages associated with skewed reaction wheel configurations, there are implementation difficulties that need to be assessed. This section identifies and analyzes potential skewed reaction wheel implementation problems.

Parallel Wheel Operation

Spacecraft maneuvers and attitude error signals are most easily expressed in a body-fixed orthogonal coordinate system. Given a policy for formulating torque required, $\vec{T}_R$, as a function of attitude error and the nominal desired maneuver, the orthogonally mounted reaction wheel inner loop command, $\dot{\vec{h}}_{BC}$, is simply:

$$\dot{\vec{h}}_{BC} = \vec{T}_R$$

Parallel operation of redundant orthogonal actuators requires a simple allocation of torque commands between colinear wheels. It is not similarly obvious in allocating the required torque to skewed wheels operating in parallel. The orientation of each of the skewed reaction wheels with respect to the body axes can be defined by a mounting matrix, C, with columns representing the direction cosines of the wheel axes in body coordinates. The skewed wheel momentum states, $\vec{h}_s$, are therefore related to wheel momentum in body coordinates as follows:

$$\vec{h}_B = C\vec{h}_s$$

To derive the skewed wheel inner loop commands, $\dot{\vec{h}}_{sc}$, it is necessary to solve the matrix equation

$$C\dot{\vec{h}}_{sc} = \vec{T}_R$$

The problem of transforming the torque required vector into inner loop commands for the skewed wheels is further complicated if parallel operation of redundant wheels is required. Parallel operation of redundant wheels is considered desirable to minimize the effect of a single wheel failure on the important near encounter experiments, and to maximize spacecraft agility for those phases. Failure of a reaction wheel at near encounter could otherwise result in the loss of attitude control at a critical stage of the mission. Redundant parallel operation of wheels would provide the control authority necessary to compensate for disturbance torques created by a failed reaction wheel. The higher spacecraft agility obtained by parallel operation of wheels would allow more experimentation to be carried out at near encounter.

The mounting matrix in the inner loop control law is of dimension 3 x n for parallel operation, where n is the number of parallel wheels. It is shown by Greville[8] that the control law solution can be defined in terms of a pseudo-inverse, i.e.,

$$\dot{\overline{h}}_{sc} = C^{\dagger}\overline{T}_R + \overline{Z}, \text{ where}$$

$$C^{\dagger} = C^T(CC^T)^{-1}, \text{ and}$$

$$\overline{Z} = (I - C^{\dagger}C)\overline{\omega}$$

$C^{\dagger}$ is the pseudo inverse, $\overline{Z}$ is orthogonal to the column space of C^T, and $\overline{\omega}$ is an arbitrary vector of rank equal to the orthogonality constraint matrix, $(I - C^{\dagger}C)$. For $\overline{\omega} = 0$, the pseudo-inverse control law minimizes the norm of the wheel torque command.

For $n = 3$, the pseudo-inverse is the inverse, as normally defined, of a matrix. The pseudo-inverse inner loop control law allows parallel operation of an arbitrary number of redundant wheels. The software required to do this, however, is more complex than the attitude control software required for orthogonal reaction wheels. The impact of the software complexity on the skewed-versus-orthogonal reaction wheel trade study is assessed later on in the paper.

Gyroscopic Cross Coupling

Gyroscopic torques act on a spacecraft when it maneuvers about an axis perpendicular to the angular momentum vectors of internal rotating parts. These torques cause excessive pointing error and power consumption. The effect of gyroscopic cross coupling is reduced by formulating the required torque in such a fashion that gyroscopic torques are predicted and thus compensated. The control law which accomplishes this task is called a decoupling control law. To derive the spacecraft decoupled control law, let us first define the system angular momentum, $\overline{H}$, as a function of vehicle moments of inertia (I_x, I_y, I_z), components of inertial angular velocity (p, q, r), and reaction wheel momentum

(h_x, h_y, h_z), all in body axes as follows:

$$\overline{H} = (I_x p + h_x)\overline{i} + (I_y q + h_y)\overline{j} + (I_z r + h_z)\overline{k}$$

The torque on the spacecraft is then

$$\frac{d\overline{H}}{dt} = \dot{\overline{H}} + \omega_x \overline{H}$$

The spacecraft's body-fixed axes are nominally coincident with a suitably defined orbit reference coordinate system. Ignoring orbital rate, which is very small for OPE missions, the linearized relationship between the spacecraft's inertial angular rates in body coordinates, and the orbit reference to body axes Euler angle rate is,

$$\begin{Bmatrix} p \\ q \\ r \end{Bmatrix} \cong \begin{Bmatrix} \dot{\varphi} \\ \dot{\theta} \\ \dot{X} \end{Bmatrix}$$

The spacecraft response to a control torque h_x, h_y, h_z, derived from the above spacecraft torque equation can be written as

$$\begin{bmatrix} I_x\ddot{\varphi} \\ I_y\ddot{\theta} \\ I_z\ddot{X} \end{bmatrix} = \begin{bmatrix} 0 & \dot{X} & -\dot{\theta} \\ -\dot{X} & 0 & \dot{\varphi} \\ \dot{\theta} & -\dot{\varphi} & 0 \end{bmatrix} \begin{bmatrix} h_x \\ h_y \\ h_z \end{bmatrix} - \begin{bmatrix} \dot{h}_x \\ \dot{h}_y \\ \dot{h}_z \end{bmatrix}$$

where the second order terms have been deleted. Thus, if a control torque is generated about the X body axis by $\dot{h}_x$, and wheel momentums h_y and h_z are none-zero, undesired spacecraft response will result from the gyroscopic torques about the Y and Z body axes. This Euler angle response differential equation can be written in matrix form as,

$$I\dot{\overline{\omega}} = M\overline{h}_B - \dot{\overline{h}}_B$$

The objective of decoupled control is to derive a wheel torque response, $\dot{\overline{h}}_B$, such that the Euler angle differential equations, $I\dot{\overline{\omega}}$, are decoupled, and attitude errors, $\overline{\varepsilon}(\varphi, \theta, X)$ are nulled. To accomplish this the reaction wheel response for orthogonally mounted wheels must be of the form

$$\dot{\bar{h}}_B = M\bar{h}_B + G\bar{\varepsilon}$$

where G is a suitable gain matrix which operates on the vector function of attitude error.

For decoupled control of wheels on skewed axes, it is necessary to find a wheel torque command, $\dot{\bar{h}}_{sc}$, such that

$$C\dot{\bar{h}}_s = \dot{\bar{h}}_B$$

This will be the case if the skewed wheel response equation is of the form

$$\dot{\bar{h}}_s = C^{\dagger}[MC\bar{h}_s + G\bar{\varepsilon}]$$

The norm of the skewed wheel momentum vector in body coordinates would not exceed the norm of the orthogonal wheel angular momentum vector. However, as noted earlier in this paper, the pseudo-inverse control law results in a weight (or torque) gain factor greater than unity for some wheel combinations. This means that decoupled control will require more power for skewed actuators. The decoupled control law for skewed wheel configurations also requires the transformation of wheel angular momenta to body axes.

Skewed Wheel Desaturation

Reaction wheels are desaturated with a mass expulsion attitude control system. Two methods of unloading reaction wheels[2] are: activating the mass expulsion thrusters for a fixed interval, with the torque-time product equal to the momentum stored in the wheel, or; applying voltage to the wheel to drive it toward zero speed, allowing the mass expulsion system to maintain attitude control while this operation is being performed.

The first approach requires that the reaction wheel momentum state be transformed into the thruster coordinate system for the skewed system desaturation control law. The second approach would be exactly the same for skewed or orthogonal reaction wheels. Therefore, assuming the second scheme, there need be no software impact for desaturation of skewed wheels.

Failure Detection

The extremely long two way communication times associated with OPE missions require that attitude control system failure detection be accomplished onboard. Certain types of reaction wheel "hard" failures are easily detected by monitoring the electrical characteristics of the actuator. Wheel failures which merely result in degraded performance are more difficult to detect. Control loop compensation and actuator nonlinearities complicate on-board failure detection which must rely on a comparison between desired and actual short period response. Failure detection maneuvers can be performed which provide data for comparison of desired and actual steady state response. Such maneuvers result in significant spacecraft attitude excursions which could disrupt communication with the spacecraft.

A failure detection technique which minimizes spacecraft attitude disturbance has been developed. This technique utilizes the pseudo-inverse orthogonality constraint matrix. The pseudo-inverse inner loop control law discussed above is of the form

$$\dot{\bar{h}}_{sc} = C^{\dagger}\bar{T}_R + \bar{Z}$$

When a failure occurs the outer loop control law provides a torque required, $\bar{T}_R$, which compensates for disturbance torques created by the failed wheel. The $\bar{Z}$ vector represents a control which can be used to modify the reaction wheel angular momentum state without imposing a torque on the spacecraft. This control vector can be utilized to detect and isolate reaction wheel failures without significantly disturbing the attitude of the spacecraft.

For the conical configuration of six redundant reaction wheels it can be shown that the orthogonality constraint matrix is defined as follows:

$$(I - C^{\dagger}C) = \begin{bmatrix} 1/2 & -1/3 & 0 & 1/6 & 0 & -1/3 \\ -1/3 & 1/2 & -1/3 & 0 & 1/6 & 0 \\ 0 & -1/3 & 1/2 & -1/3 & 0 & 1/6 \\ 1/6 & 0 & -1/3 & 1/2 & -1/3 & 0 \\ 0 & 1/6 & 0 & -1/3 & 1/2 & -1/3 \\ -1/3 & 0 & 1/6 & 0 & -1/3 & 1/2 \end{bmatrix}$$

The following sequence of $\vec{\omega}$ vectors is implemented for fault isolation testing.

$$\omega_1 = \begin{bmatrix} \omega_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}; \quad \omega_2 = \begin{bmatrix} 0 \\ \omega_2 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}; \quad \omega_3 = \begin{bmatrix} 0 \\ 0 \\ \omega_3 \\ 0 \\ 0 \\ 0 \end{bmatrix}; \quad \omega_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \omega_4 \\ 0 \\ 0 \end{bmatrix}; \quad \omega_5 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \omega_5 \\ 0 \end{bmatrix}; \quad \omega_6 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \omega_6 \end{bmatrix}$$

If no failure has occurred, no significant disturbance would be created on the spacecraft by these six zero-net-torque output tests. If a wheel has failed, a spacecraft disturbance would be sensed by the attitude reference system when four of the six test control vectors were applied. The spacecraft disturbance generated by the test control, when a failure was present, would be minimal because the pass/fail criteria merely involves sensing a disturbance, with no comparison of the magnitude of the disturbance with a predicted disturbance required. For the failure detection test sequence defined by the six control vectors above, the failed wheel can be isolated by observing the two test controls for which the satellite is not perturbed. The simple logic shown in Table 2 is used to isolate the failed wheel.

Table 2

SINGLE WHEEL FAILURE ISOLATION LOGIC

Failed Wheel	Inertial Hold Sequence
1	3,5
2	4,6
3	1,5
4	2,6
5	1,3
6	2,4

There is no equivalent minimum spacecraft disturbance failure detection testing for orthogonal redundant reaction wheels. The pseudo-inverse failure isolation does, however, represent an additional software complexity.

COMPUTATIONAL REQUIREMENTS COMPARISON

The apparent negative aspect of the skewed reaction wheel configurations is increased computational complexity. The more complex software requirements for implementation of the skewed configurations detract from their inherent reliability, so that a measure of the increase in computational capability must be assessed prior to the selection of a skewed configuration. The reliability implication of the added software requirement is of particular interest, since reliability is the most difficult OPE design requirement.

The approach taken in this evaluation has been to model as parallel as possible a computer implementation for both the orthogonal and skewed redundant configurations. Their inherent differences have made completely parallel modeling impossible in some areas (such as fault isolation and redundancy failure switching) for which the orthogonal configurations do not provide a counterpart. Modeling in these areas has purposely been such that computations are a worst case for the skewed configurations.

For purposes of concrete comparison, a six wheel dual redundant orthogonal configuration has been taken as a standard. The six wheel skewed configuration was studied to the same detail. Extrapolation of the results for these configurations has been made for configurations with more redundant wheels.

Control System Modeling

For the orthogonal configuration model, the angular momentum is input to the computer each cycle for each of the six wheels and summed, for each set, into angular momentum components along the three body axes. Operating on these as dictated by the control law, a resulting command is obtained and output to one or both of the two wheels along each of the three axes.

In the skewed configuration, the angular momentum is input to the computer each cycle from each of the six wheels and retained as a 6 x 1 matrix, $\overline{h}_s$. This is converted to body coordinates by means of the six sets of direction cosines (the 3 x 6 matrix, C). The same control law is used as for the orthogonal configuration, except that the control law computations are followed by the pseudo-inverse transformation to obtain the $\dot{\overline{h}}_{sc}$ command for each of the active wheels in the configurations. The $\dot{\overline{h}}_{sc}$ commands are output to the six wheels, null commands going to inactivated or failed wheels which would previously have been switched to ignore them.

An implementation was assumed which would incorporate either or both of the reaction wheels along each of the three orthogonal axes. This is roughly the same versatility as allowing any (three or more) of the six skewed wheels to be operating at any given time. In the skewed configuration, however, this versatility can be provided by performing exactly the same calculations and logic each cycle, even when operating modes have changed. The reason for this is that nonoperating wheels will have zeros stored in the column of the direction cosine matrix associated with their orientation at the time of switching. This will result in the command for that wheel being zero at each subsequent calculation.

Since in the worst case timing problem all six wheels may be operating, it would actually increase the timing to provide separate switching, rather than to go through all the calculations for lesser operating modes. It would greatly increase the execution time for the orthogonal configuration to employ a similar technique, since no transformation from body axes to wheel orientation is required for that configuration. Thus, the additional computational complexity of the skewed implementation buys a simplification in the wheel switching logic. Fig. 6 illustrates the control law implementations for the two configurations.

The wheel fault isolation mechanizations were implemented for both configurations. In the implementation of the orthogonal configuration, failure isolation results in the switching of the failed wheel to ignore further commands, and the saving of the failed wheel identification for use in future command allocations. For the skewed configuration these actions are performed in addition to the execution of a defensive failure mode switching technique following the second wheel failure. There is no analogous capability to be accommodated for orthogonal configurations.

Flow diagrams for the respective fault isolation and wheel switching logic are included in Fig. 7.

Computer Implementation

To obtain an assessment of computer cycle time and computer storage requirements, computer programs were developed for what is felt to be a very typical aerospace computer. This hypothetical computer was assumed to have double precision fixed-point arithmetic capability, an A and Q register, and up to three index registers, in addition to providing an indirect addressing capability.

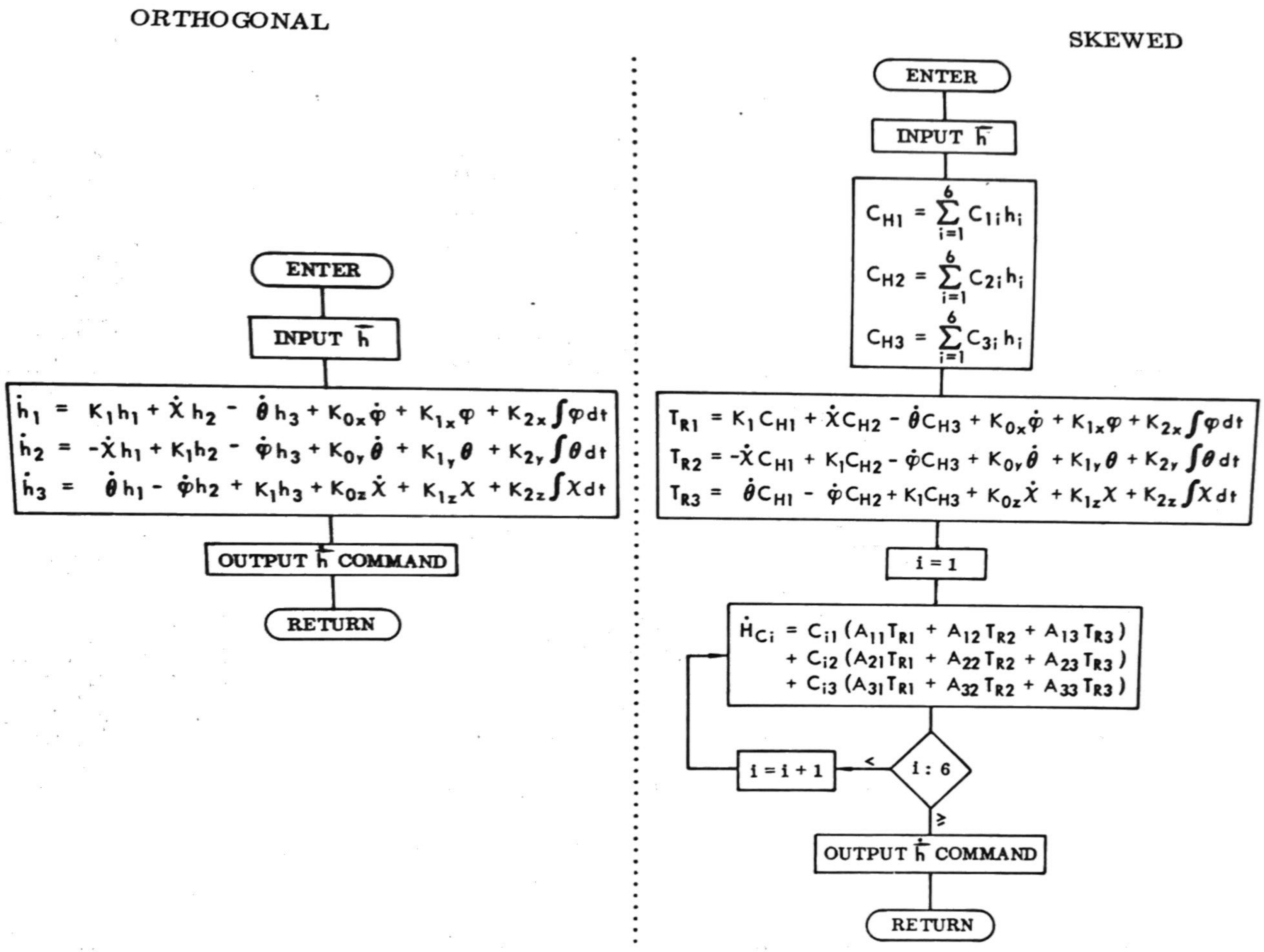

Fig. 6 CONTROL LAW PROGRAMS FOR ORTHOGONAL AND SKEWED CONFIGURATIONS

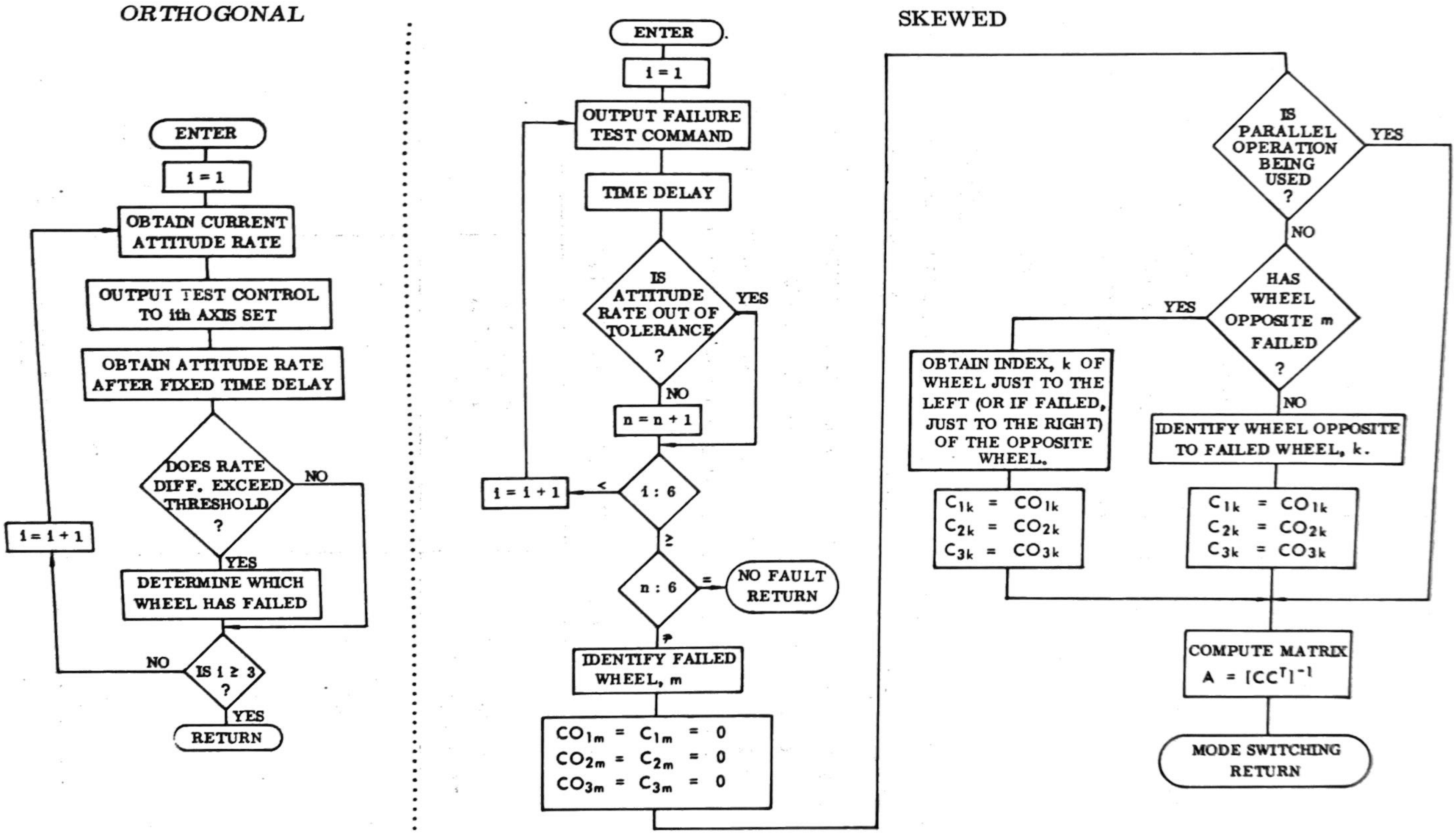

Fig. 7 FAULT ISOLATION AND WHEEL SWITCHING LOGIC FOR ORTHOGONAL AND SKEWED CONFIGURATIONS

program optimizing techniques were employed for both configurations. Program execution time was minimized for the respective control loops, and data storage was minimized in the fault isolation and failure switching programs. For example, in the skewed implementation, a matrix $A = (CC^T)^{-1}$ was defined and calculated only at the time of operational mode switching to reduce to an absolute minimum the number of multiplications required in the control loop. The pseudo-inverse control law requires the pseudo-inverse of all possible combinations of three or more columns of the skewed wheel direction cosine matrix. A software trade study was conducted to determine whether computation or storage of the matrices minimized overall data storage. Computation of the matrices required a matrix inversion subprogram. It was determined that computation of the inverses reduced storage requirements from 486 to 36 words.

Comparative Results

For the programs that were coded, computer storage and cycle time requirements were obtained and interpreted for the computer reliability implications.

Computer storage requirements for the orthogonal and skewed reaction wheel configurations are respectively 175 and 420 locations. Table 3 contains a breakdown of storage requirements into data and instruction storage for the control loop, failure isolation, and redundancy failure switching programs.

TABLE 3

COMPUTER REQUIREMENTS COMPARISON

Program Storage \ System	Orthogonal Redundant Configuration	Skewed Redundant Configuration
Control loop:		
Instr.	62	100
Data	40	69
Gyro Failure:		
Instr.	58	164
Data	15	42
Defensive Degraded Mode Switching:		
Instr.	Not Applicable	23
Data		22
Total	175	420

The additional 245 locations required to implement the skewed configuration software will not require additional address decoding or memory accessing logic. The reliability impact is therefore proportional to the ratio of 245 to the total number of computer locations otherwise required for the OPE missions. This ratio is felt to be a small number.

Cycle time requirements were obtained by assuming that the control law computations must be performed once each 100 milliseconds and that $X\%$ of the total computational capability of the computer is to be dedicated to attitude control for worst case computer time loading.

The number of each type of instruction to be executed in the control loop was tabulated. All instructions were assumed to require the same execution time, except for the multiply and divide instructions which were assumed to require η times as long. The resulting number of unit executions per cycle for the orthogonal and skewed configurations are respectively $N_o = 169 + 18\eta$, $N_s = 238 + 63\eta$. Typically, η is about five so that $N_o = 259$ and $\Delta N = N_s - N_o = 294$.
The equation for required unit execution time in microseconds is

$$\tau_u = \frac{10^3 X}{N_o + \frac{X}{100}\Delta N} = \frac{X}{0.259 + 0.00294 X}$$

There are two ways to evaluate the impact of ΔN on the OPE missions. If it is assumed that τ_u is given, the impact of ΔN on the percentage of the computational capacity required by the attitude control subsystem can be determined. Or, if it assumed that X is specified, the impact of ΔN on the unit execution time can be determined. Taking the latter approach, if X is taken as 10%, then τ_u must be decreased from 38.7 to 34.6 μs. With subsystems being developed simultaneously, however, the first approach is probably the most acceptable. If τ_u is given as 5 μs, which is a reasonable add time, the control subsystem percentage allocation would need to be further increased by 1.4%.

For skewed configurations, involving more than six wheels, the primary impact on data storage will be in the redundancy failure switching since they require more levels of switching logic than that required for the six wheel configuration. A switching philosophy probably could be developed to give full reliability potential, and at a large saving in data storage.

The cycle time impact will be primarily the additional multiplications required in the control loop: For each additional wheel, there will be six additional multiply operations and N_s will increase by approximately 60 unit executions.

The data presented above indicate that if either a low-speed ultra reliable computer, or one with more typical computational capability is used, the impact on the computer reliability due to the increased computational complexity of the skewed reaction wheel configurations is not appreciable. The skewed reaction wheel configurations do not require a basically different computer; at most one which is ten per cent faster (or alternatively, has more time allotted to attitude control) and contains five per cent more storage locations. This is especially the case since software is usually sized ten to twenty per cent on the high side to allow a margin of safety, and the impact of the more extensive computations is much less than this.

CONCLUSIONS

Skewed reaction wheel failure mode switching logic will minimize the probability of having only adjacent wheel combinations available at the near encounter phases of the OPE missions. The skewed wheel configurations can, therefore, be utilized to greatly improve attitude control system reliability, for a slight wheel weight increase over an orthogonal configuration with equal performance. Control law, failure isolation, and redundancy switching software is more complex for the skewed redundant reaction wheel system. The impact of this software complexity on the computer timing and storage requirement is quite small. The computer reliability reduction due to skewed rather than orthogonal wheels for attitude control will, therefore, also be small.

It is concluded that skewed reaction wheels should be utilized for OPE spacecraft to enhance long life, endurance, and operability.

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