## Replacing Point Particles with Electric and Gravitational Charge Distributions

Solution of the Poisson equation appropriate to an amount of charge in free space when the proper boundary condition is imposed at the origin as well as at infinity results in unique expressions for the charge density, total charge, potential, and field strength.

## implied alternative constructs

Here we list the expressions for all of the associated theoretical constructs that result from the alternative solution.

$$
\begin{array}{ll}
\rho(r)=\frac{\alpha \mathrm{q}_{0}}{4 \pi r^{4}} \mathrm{e}^{-\alpha / r} & \mathrm{q}(r)=\mathrm{q}_{\mathrm{o}} \mathrm{e}^{-\alpha / r} \\
V(r)=\mathrm{q}_{\mathrm{o}}\left(1-\mathrm{e}^{-\alpha / r}\right) / \alpha & \mathrm{E}(r)=\mathrm{q}_{\mathrm{o}} \mathrm{e}^{-\alpha / r} / r^{2}
\end{array}
$$

These all involve the inverted exponential factor, which has not been present in electrostatic and gravitational theoretical expressions to date. We will provide graphic illustrations of each. It is instructive to consider the extent to which this inverted exponential factor constrains the various constructs. As we will see, the behavior of these constructs in the vicinity of $r=\alpha$ is very different than their behavior at larger as well as at smaller distances from the center of charge. It is instructive to consider the magnitude of this factor for various values of $r / \alpha$.

| range | minimum | factor |  |  |  |
| :--- | :---: | :--- | :---: | :--- | :--- |
| maximum |  |  |  |  |  |
| if $\mathrm{r} / \alpha>1000$ | 0.999 | $<$ | $\mathrm{e}^{-\alpha / \mathrm{r}}$ | $<$ | 1.0 |
| if $\mathrm{r} / \alpha>100$ | 0.99 | $<$ | $\mathrm{e}^{-\alpha / \mathrm{r}}$ | $<$ | 1.0 |
| if $\mathrm{r} / \alpha>10$ | 0.90 | $<$ | $\mathrm{e}^{-\alpha / \mathrm{r}}$ | $<$ | 1.0 |
| if $\mathrm{r} / \alpha=1$ | 0.3679 | $=$ | $\mathrm{e}^{-\alpha / \mathrm{r}}$ | $=$ | 0.3679 |
| if $\mathrm{r} / \alpha<1 / 2$ | 0.00 | $<$ | $\mathrm{e}^{-\alpha / \mathrm{r}}$ | $<$ | 0.1353, |
| if $\mathrm{r} / \alpha<1 / 5$ | 0.00 | $<$ | $\mathrm{e}^{-\alpha / \mathrm{r}}$ | $<0.00674$ |  |
| if $\mathrm{r} / \alpha<1 / 7$ | 0.00 | $<$ | $\mathrm{e}^{-\alpha / \mathrm{r}}$ | $<0.0009$ |  |
| if $\mathrm{r} / \alpha<1 / 10$ | 0.00 | $<$ | $\mathrm{e}^{-\alpha / \mathrm{r}}$ | $<0.0000454$ |  |
| if $\mathrm{r} / \alpha<1 / 20$ | 0.00 | $<$ | $\mathrm{e}^{-\alpha / \mathrm{r}}$ | $<2.061 \times 10^{-9}$ |  |
| if $\mathrm{r} / \alpha<1 / 100$ | 0.00 | $<$ | $\mathrm{e}^{-\alpha / \mathrm{r}}$ | $<3.72 \times 10^{-44}$ |  |

All of the above analyses are compatible with all theoretical considerations and experimental results. It has consistent application to fundamental particles. In addition to an escape from the necessity of 'point' particles and the associated singularities, we will show that the revised expression for the potential actually accommodates the formulation of classical electrostatics (as well as gravity) without endorsing action-at-a-distance that has been suspect since its inception.

This feature derives from the nature of option 3 above for which there is no distance dependence of the potential, leaving only the amount of charge at the distance as determining the potential and therefor the experimentally observable force. The gradient in the derivation above can now be taken without compunction to evaluate both the potential and charge density at the same locations throughout space including the boundaries at the origin and at infinity as we will demonstrate.

## implied difference in the conjectured potential

How closely the alternative potential field can be characterized as the inverse of the separation of one charge from another is significant. The form that was accepted (albeit without experimental verification at the submicroscopic level applicable to fundamental particles) for centuries is shown as the upper curve in figure 1.1. The figure provides $\log$ plots of the two alternative forms for the potential. When applied to the charges of up and down quarks the new conjecture, shown below the traditionally accepted form becomes virtually indistinguishable at distances greater than $10^{-14}$ cm . By extending the domain of the plot out to a distance of $10^{-8} \mathrm{~cm}$ in the $\log$ plot of figure 1.2 , we see that there is virtually no difference at all due to this change in hypothesis.


Figure 1.1: Potential as a function of radial distance of a distribution (for $r<2 x 10{ }^{13} \mathrm{~cm}$ )
It is through the polynomial expansion of the exponential that we find agreement with the conventional theory. The series expansion of an exponential is given by the following:
$e^{-x}=1-\frac{x}{1!}+\frac{x^{2}}{2!}-\frac{x^{3}}{3!}+\frac{x^{4}}{4!}-\cdots$
Thus, we arrive at an approximation for $\mathrm{V}(r)$ appropriate whenever $r^{2}$ is appreciably greater than $\alpha$ as follows:
$\mathrm{V}(r)=\mathrm{q}_{0}\left\{\frac{1 / r}{1!}-\frac{\alpha / r^{2}}{2!}+\frac{\alpha^{2} / r^{3}}{3!}-\frac{\alpha^{3} / r^{4}}{4!} \cdots\right\}$

The significance of this alternative is that it completely avoids singularities. The charge/mass previously embodied in 'point' particles can now legitimately be associated with continuous distributions of charge/mass instead.


Figure 1.2: Log of potential as a function of radial distance of a distribution (for $r<1 \times 10^{-8}$ cm )

## erroneous conventional wisdom

Although in experimental agreement, what we have demonstrated is at considerable odds with conventional wisdom. We know there is a traditionally-accepted explicit dependence of the potential on distance, based upon the assumption of point charges, which is:
$V(r)=\sum_{\mathrm{i}} \mathrm{q}_{\mathrm{i}} /\left|\mathrm{r}-\mathrm{r}_{\mathrm{i}}\right|$
It even makes sense to speak of such an ensemble of charges in terms of a 'continuous function' $\mathrm{q}(r)$. It is comprized of indivisible point particles with the gradient zero everywhere except at 'points' given by $r_{i}$ where that value is infinite. Thus, for a single indivisible particle the assumed potential has been:
$V\left(r_{\mathrm{i}}\right)=\mathrm{q}_{\mathrm{i}} / r_{\mathrm{i}}$, for $r_{\mathrm{i}}>0$.
here $\mathrm{q}_{\mathrm{i}}$ is the total charge of one indivisible point particle. This is illustrated in figure 1.3, where the particle is assumed to be located at the origin so that $r_{\mathrm{i}}=\left(x_{\mathrm{i}}^{2}+y_{\mathrm{i}}^{2}+z_{\mathrm{i}}^{2}\right)^{1 / 2}$. The plot avoids the origin where the point particle is located because the value of $V(r)$ is infinite at that point. (Notice that $y$ only ranges from 1 (not zero) to 10 in the plot.) The potential is specified for a location that is removed a distance $r_{i}$ from the particle in the above expression. It has no bearing whatsoever on anything physical at the location $\mathbf{r}_{\mathbf{i}}$. The potential in this case expresses merely a geometrical relation between the source of the action at the origin and action that takes place at the location $\mathbf{r}$, i.e., it assumes action-at-a-distance.


Figure 1.3: Potential as conventionally conceived
In this previously accepted approach the charge is all concentrated at the single point at the origin where it possesses an infinite value, but whose 'encapsulated' charge about the origin is $q_{i}$ as shown in figure 1.4. Clearly the Dirac delta function is required to shore up the mathematics of this approach. Discussions of the delta function predominate in introductions to the treatment of electrostatics and gravitation because without justification for how the discontinuities are to be handled, the theory would be invalid.


Figure 1.4: Conventionally conceived charge concentration

## alternative conjecture in the current investigation

The conjecture we have introduced involves particles comprized of continuous charge densities such that the accumulated charge, $\mathrm{q}(r)$ is a continuous differentiable function. In this case the potential associated with the particle is related to encapsulted charge as:
$V(r)=\left(\mathrm{q}_{\mathrm{o}}-\mathrm{q}(r)\right) / \alpha$
Here both $\mathrm{q}_{0}$ and $\alpha$ are constants defining the indivisible charge density. This potential is illustrated in figure 1.5. So in this approach the solution to the Poisson differential equation is:


Figure 1.5: Potential, $(q-q(r)) / \alpha$ in the current conjecture
Figure 1.6 illustrates the charge density; the potential, charge density, and all other constructs are spherically symmetric about the center of the particle. The total charge out to a given radius and the field strength are illustrated in figures 1.7 and 1.8.


Figure 1.6: Charge density in the current conjecture


Figure 1,7: Total charge in a sphere of radius $r$


Figure 1.8: The electric field strength (force) at a distance r from the center of the distribution in the current conjecture

## the intersection of electricity and gravitation

We have associated these two properties of particles as integrally related by insisting on equivalence of rest mass and electrostatic self-energy. The Poisson distribution is the only viable distribution of charge throughout all space that implies a precise quantity of each property while meeting reasonable boundary conditions at the origin and at infinity. But our treatment so far has focused on the electrostatic aspect of the particle and not gravitational. In addition to attributing mass to electrostatic self-energy, we assume that the mass of any object in and of itself is associated with a gravitational force field of a similar form to that of the electrostatic force.

To warrant common treatment with the Poisson equation, requires further consideration of the unilaterally attractive nature of gravity. One approach is to assume two separate force fields surrounding basic particles, one electrostatic, the other gravitational. Another approach would be for gravitation to be a residual aspect of the electric field observable only when electrical forces are neutralized. The differences will in retrospect seem to be a distinction hardly worth mentioning.

The Poisson equation used in the derivation of the distribution of charge is universally accepted as applying equally to gravitation. We make no excuses for the approach we applied in the previous chapter nor to applying it now to gravitation. What remains to explain is how does this approach pertain to 'mass' as equivalent to the self-energy of electrical charge, what is the gravitational charge $\sqrt{\mathrm{G}} \mathrm{m}$, and how does that contribute to the indivisibility of particles?

## gravitational enforcement of indivisibility

The form of the Poisson distribution of charge with its implied charge-related self-energy applies also to the distribution of associated gravitational mass whose force field may cancel the electrostatic field near the origin but leaves it essentially unaltered at appreciable distances. That is the reason that fundamental charge distributions are indivisible and act simply as particles.

While it is impressive that the negative energy in the gravitational field could cancel that of the electrostatic field whose self-energy is the basis for there being any mass at all to contribute to gravitation, variance of the gravitational charge distribution need not be that small to effect indivisibility. Any value less than the electrostatic variation would suffice to produce indivisibility. In any case, the magnitude of that gravitational field is miniscule by comparison to the value of the electrostatic field at all distances for which net charge is appreciable. In figure 2.3 we illustrate the comparison of the force field strengths for $q_{m}$ much less that $q_{e}$ and for various values of $\alpha_{m}$ less than $\alpha_{e}$ none of which are anywhere near as small as those that are actuality realized.

Notice that there is a pivotal distance at which the shorter-range gravitational force exceeds the electrical force. To illustrate this more fully requires the use of log scale plots of the absolute values of the respective forces. In figure 2.4 we illustrate domains of significant influence of gravitational cohesion and electrostatic repulsion forces for properties that we associate with up and down quarks. These properties as are follows:

Gravitational charge of the down quark: $2.6113583 \times 10^{-32}$
Gravitational variance of the down quark: $3.6349321 \times 10^{-17}$
Gravitational charge of the up quark: $2.2472527 \times 10^{-28}$
Gravitational variance of the up quark: $1.6895847 \times 10^{-20}$
This hypothetical alternative of the gravitational variance of the up and down quarks assumes that $\alpha_{\mathrm{m}}=\sqrt{\mathrm{G}} \alpha_{\mathrm{c}}$. It avoids the ineffectuality of $\alpha_{\mathrm{m}}=\alpha_{\mathrm{c}}$ as well as the perhaps too extreme binding of a second hypothetical alternative we will consider where $\alpha_{\mathrm{m}}=\sqrt{\mathrm{G}}(\mathrm{m} / \mathrm{q}) \alpha_{\mathrm{c}}$.


Figure 2.3: Combined electrostatic and gravitational forces


Figure 2.4: Combined electrostatic and gravitational forces of up and down quarks with the difference in variance between electrical and gravitational field variances being $\sqrt{\boldsymbol{G}}$

The rationale for one or the other (or a third) alternative for the gravitational variance centers around the matching the observed ability of quarks to combine without absorption into single Poisson distributions.

