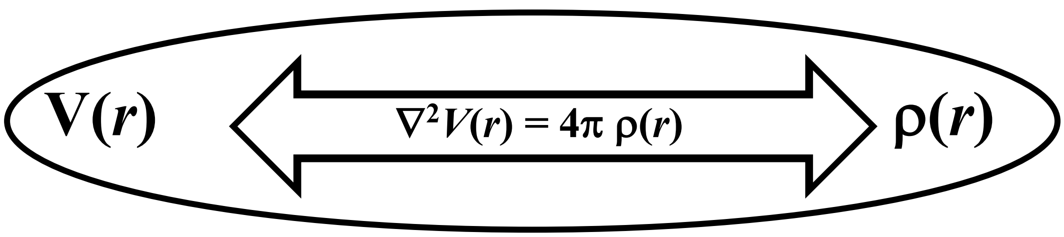
Fundamental Particles and Inverse Square Law Forces

There are no inverse square law forces; rather, there are forces with an inverse square factor that comes to predominate at experimentally determined distances from the centers of force.

Centuries of scientific investigations have exposed the particulate nature of matter comprised of indivisible particles possessing electronic charge and gravitational mass.[1] Included is all 'ordinary' matter tabulated in the periodic table of the elements. Constituent particles of the one hundred odd elements are few: protons, neutrons, and electrons, with a further breakdown into quarks – primarily 'up' and 'down' quarks. Thus, this ordinary class of matter involves fundamental particles, each possessing a discrete amount of charge and (rest) mass. The effects of these indivisible particles cannot be circumscribed and thereby neatly restricted in space.

As a part of theoretical and experimental investigations undertaken over the last few centuries, forces between particles possessing charge and mass have been formulated and incorporated into a classical field theoretic expression that encapsulates the experimental evidence. In this discipline forces associated with their electric charge (or gravitational mass) can be determined by solving what is called the 'Poisson boundary value problem', i.e., a formulation of a second order partial differential equation that relates the potential energy field (a function) $V(\mathbf{r})$ to the charge (or mass) density distribution (also a function) $\rho(\mathbf{r})$. Valid solutions must satisfy specified conditions at the boundary of the volume of space under consideration. For example, the density of a distribution of charge in open space must tend to zero at an infinite distance from the center of the distribution and involve a finite density of charge and mass at each point within that domain.



Logical relation (at location* r*) established by the Poisson equation

If indivisible charge and mass were to exist only at a point as typically propounded of material particles, there would have to be *other* forces to avoid implosion into singularity or infinite repulsion keeping like electric charges from joining.[2] It seems unlikely that any 'real object' of nature could exist as a mere mathematical point. There are many reasons why the established approach is not beyond dispute. On the other hand, an extensive indivisible distribution would have to exist force-free, i.e., no portion could experience any force relative to the rest of the distribution. Any adequate resolution of associating charge and mass distributions with indivisible particles formulated as a Poisson boundary value problem must resolve all such issues.

theoretical background in classical field theory

Newton's gravitation and Coulomb's electrostatics gave rise to extremely similar theoretical formulations of the associated forces between material objects. There are precursor vector

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[1] Hypotheses of 'dark matter' have more recently obscured the issue somewhat, but we will exclude such vaguely conceived 'exotic' matter from the discussion here as well as the distinction between inertial and gravitational mass.
[2] Indeed, the somewhat *ad hoc* 'strong force' addresses this issue.

differential equations and associated proven theorems that ultimately give rise to a synthesis in the Poisson equation shown above. These prerequisite equations and related theorems apply in the domain of mathematics. In order to properly apply them to physical reality, it had to be shown that the abstract mathematical constructs and associated assumptions were isomorphic to physical quantities subject ultimately to experimental observation.

The function $V(r)$ in the Poisson equation above is defined as the 'potential energy' of a 'unit test particle' at the location $\mathbf{r}$. The distribution of material substance (whether charge of mass) $\rho(r)$ is the density of that substance at location $\mathbf{r}$. The use of r in place of $\mathbf{r}$ is to indicate the spherical symmetry assumption implicit in the functions when applied to their contribution to an indivisible particle. Values of $V(r)$ are not experimentally observable; rather, they are inferred from experimental measurements of $\mathbf{E}(r)$ which is the force on a unit particle at $\mathbf{r}$. Thus, $V(r)$ is defined to be no more nor less than is consistent with $\mathbf{E}(r)$, which equals the negative of its gradient $\nabla V(r)$ – the derivative of $V(r)$ with respect to r the distance from the center of a spherical particle. The expression $\nabla^2 V(r)$ is the 'divergence' of $\mathbf{E}(r)$ or the second derivative of $V(r)$ with respect to r. With these caveats, the supporting theorems lead ultimately to the Poisson equation.

This Poisson partial differential equation in consort with associated boundary conditions incorporates in total the experimentally verified relation between the density of a distribution of charge or mass $\rho(r)$ and the force field $\mathbf{E}(r)$ or potential energy $V(r)$ in accordance with the classical theories of electrostatics and gravitation respectively. The role of $V(r)$ (determined solely as consistent with its gradient) is of special interest with regard to allowing some latitude in satisfying alternative boundary conditions. The solutions determine stable (force free) conditions for charge or mass distributions. What is implicit is that the total amount of energy (or rest mass) as well as the charge encapsulated in a distribution are co-determined with the functional form of the resulting distribution. To date that implication seems to have remained undiscovered. One must ask, why?

boundary value issues

A basic understanding of the formulation of the Poisson equation is necessary. Difficulties arise with regard to determining the extent of the domain for which solutions will apply and the conditions enforced at these extremities. These considerations determine whether any, or too many, solutions may exist. So, one must carefully define domain boundaries and select the type of boundary conditions that properly apply within the mathematical framework to assess whether a unique solution exists.

There are various legitimate qualifications for boundary conditions. The potential being specified throughout the boundary (e.g., the infinite spherical surface for a domain including all of space) refers to *Dirichlet* boundary conditions. The field vector component normal to the boundary surface (outward force) being defined everywhere on the boundary refers to *Neumann* boundary conditions. A *Cauchy* boundary condition is one for which *both* the Dirichlet *and* the Neumann conditions are specified on all boundaries.

It should be noted that the general treatment of electrostatics addresses variously shaped and configured conductors which constitute boundaries of a domain throughout which a solution is sought. But in regard to indivisible particles, we are investigating distributions of charge or mass in open space, which we treat as a spherical region of infinite extent. Spherical symmetry, like so much in actual problem solution, involves approximation of the variations of parameter values at extreme removes from the center of a material distribution of interest. In dealing with an isolated quark whose domain of significant impact is less than 10^{-12} cm, a spherical region of a centimeter radius can define a domain with parameter values equivalent to those at an infinite remove.

Typically the only boundary considered in such situations is the spherical surface at $r = \infty$, or whatever remove provides an equivalent boundary. However, unlike this usual treatment, any proper handling of such situations necessitates the origin being treated as a boundary as well – negative values of *r* clearly are not within any 'real' bounded domain. So the origin must be considered a boundary to avoid the singularity at that point which has traditionally just been accepted as collateral damage. Since we will deal with spherical symmetry, the only symmetry is with regard to radial distance from such a center, which means that boundaries must be defined as existent at both $r = 0$ and at $r = \infty$. As we will see, using these boundaries and proper conditions, rather than leaving properties at the origin unspecified, has an extremely significant impact. It is a precondition for eliminating the difficulties of a singularity and action at a distance.

Unlike in the traditional approach, for which the potential is assumed to be viable everywhere (including the origin, at which point it is infinite), a total amount of energy associated with a given distribution can be defined. This total (self) energy can then properly be associated with the rest mass of the distribution/particle. Thus, with the proper boundary conditions, the solution of the associated Poisson equation is a distribution with given amounts of both charge (or gravitational mass) and self-energy, matching the properties of an associated particle.

problems with the inverse square law forces for distributed substance

We begin with the assumption that the field strength $\mathbf{E}(r)$ (force field) is the negative gradient of a potential function $V(r)$ for spherically symmetric distributions of continuous substance. Let the parameter $q(r)$ be the amount of substance within the sphere of radius *r* – equal to the integral of $\rho(r)$ throughout the sphere. It is usual as an aspect of the point particle proposition to assume $q(r) = q$ (total amount of substance) for all values of $r > 0$, but consider what happens if we do not make that assumption. The vector field strength is outwardly directed such that we can replace the vector quantities with scalar quantities, $E(r) = q(r) / r^2$ being experimentally warranted at distances away from the singularity. Continuing with this line of reasoning, $V(r) = -q(r) / r$. Thus, with the field strength determined as the negative gradient of the potential function, one should expect.

$$E(r) = -\nabla V(r) = -\frac{dV(r)}{dr} = -\frac{d}{dr}\left[q(r) / r\right]$$
$$= q(r) / r^2 \; - \frac{1}{r}\frac{d}{dr}q(r)$$

That this second term must be zero is an inconsistency in the traditional treatment.

An ensemble of point charges within a given radius does *not* provide a differentiable charge in which case the second term in the final expression is undefined. Thus a viable solution required settling for $q(r) = q$, independent of *r*. The differential of q vanishes by definition, an unsatisfactory compromise limiting the solution domain to *r* greater than zero:

$$E(r) = q / r^2 \text{ if } r > 0.$$

However unsatisfying, this incomplete formulation is the most direct experimental ratification of theory. It is associated with Newton's and Coulomb's inverse square law forces for which the functionality is *exclusively* an inverse square all the way to the origin where it is undefined.

The annoyance of having to explicitly omit a mathematically implied term from an accepted formula ranks up there with Einstein's insertion of an alien term in a mathematically sound Poisson equation, neither with just cause. One cannot logically accept the legitimacy of the derivation of an equation and then omit or insert terms willy-nilly that were not addressed in its derivation. The suspect singularity at the origin and concept of action-at-a-distance are ramifications of ignoring a critical term in the most basic equation of the theory.

resolution of the singularity issue

Poisson boundary value problems are, to be solved for the potential. If we limit the constraints on the potential to its experimental and theoretical imperative, the only requirements on $V(r)$ are that it be the integral of the field strength on a unit charge being brought in from an infinite remove, and that its boundary conditions be met. So, let us begin by assuming the potential to be dependent on both the functionality of the distribution, i.e., $\mathrm{q}(r)$ which in this case is the amount of the distribution contained within the radius r, as well as an explicit dependence on the distance from the center of the distribution: $V(r) = -\,\mathrm{q}(r)\,/\,r$. Thus, we analyze the experimentally testable field strength aspect of the functionality of $V(\mathrm{q}(r), r)$, namely the gradient which is:

$$\nabla V(\mathrm{q}(r), r) = \frac{\partial V(\mathrm{q}(r), r)}{\partial \mathrm{q}(r)} \cdot \frac{\mathrm{dq}(r)}{\mathrm{d}r} + \frac{\partial V(\mathrm{q}(r), r)}{\partial r}$$

Experimental observations of the field strength indicate that there is but one term and thus, one or the other of these terms must be zero. This means that one of the following must be true. We address these possibilities in order:

(1) There is no explicit dependence of V on $\mathrm{q}(r)$:

$$\nabla V(\mathrm{q}(r), r) = \frac{\partial V(r)}{\partial r} = -\frac{\mathrm{e}^{\mathrm{C}}}{r^2} = -\frac{\mathrm{q}}{r^2} \Leftarrow V(r) = \frac{\mathrm{q}}{r}$$

Where C is a constant of integration chosen to be q (total amount of substance).

(2) There is no explicit dependence of $\mathrm{q}(r)$ on the distance r:

$$\nabla V(\mathrm{q}(r), r) = \frac{\partial V(\mathrm{q}, r)}{\partial r} = -\frac{\mathrm{q}}{r^2} \Leftarrow V(\mathrm{q}, r) = \frac{\mathrm{q}}{r}$$

(3) There is no explicit dependence on distance from the center, r:

$$\nabla V(\mathrm{q}(r), r) = \frac{\partial V(\mathrm{q}(r))}{\partial \mathrm{q}(r)} \frac{\mathrm{dq}(r)}{\mathrm{d}r} = -\frac{\mathrm{q}(r)}{r^2}$$

Then we attempt a solution to this equation of the form: $V(\mathrm{q}(r)) = \mathrm{C}_2 + \mathrm{C}_1\,\mathrm{q}(r)$, from which we obtain a solution for $\mathrm{q}(r)$ as follows:

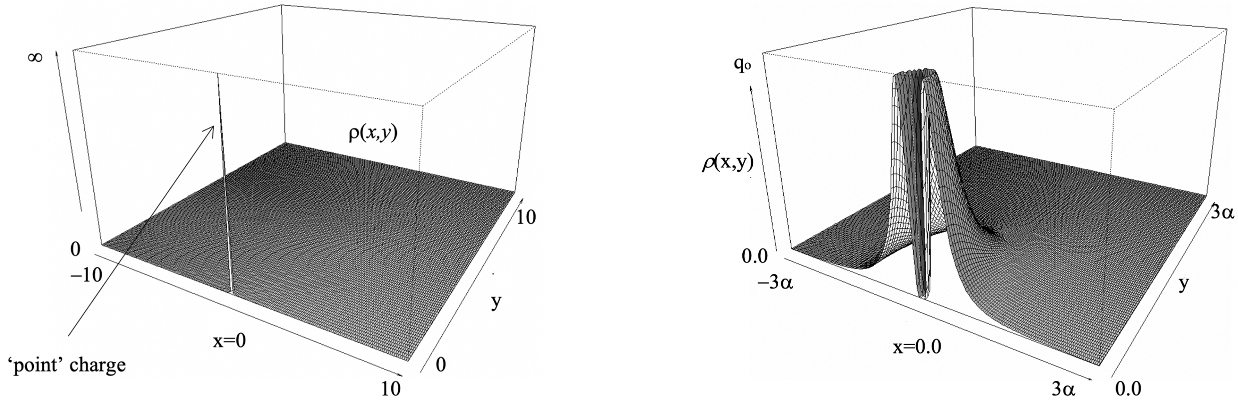
$$\mathrm{C}_1 \frac{\mathrm{dq}(r)}{\mathrm{d}r} = -\frac{\mathrm{q}(r)}{r^2} \rightarrow \frac{\mathrm{dq}(r)}{\mathrm{q}(r)} = \frac{\mathrm{d}r}{\mathrm{C}_1 r^2} \rightarrow \ln \mathrm{q}(r) = -\,(1\,/\,\mathrm{C}_1\, r) + \mathrm{C}_2$$

Finally, by taking the exponential of both sides we obtain:

$$\mathrm{q}(r) = \mathrm{q}\ \mathrm{e}^{-\alpha / r}$$

Here, the constants are assigned as follows: $\mathrm{q} \equiv \mathrm{e}^{\mathrm{C}_2}$ and $\alpha \equiv (1/\mathrm{C}_1)$.

The implied differences in charge/mass distribution when the Poisson boundary values are properly assigned are illustrated in the figures below.



Conventionally conceived charge density ***Charge density in the current conjecture***

Implied differences in charge/mass density with current conjecture

Both the solutions (1) and (2) above and solution to the Laplace equation (the homogeneous version of the Poisson equation, ρ(*r*) zero everywhere) violate the boundary condition requiring a finite value of *V* at the origin and therefore are not valid solutions. We, therefore, must accept option (3), acknowledging that in this case the integral expression for potential becomes:

$$V(r) = (\, q - q(r)\,) / \alpha = q\,(1 - e^{-\alpha / r}\,) / \alpha$$

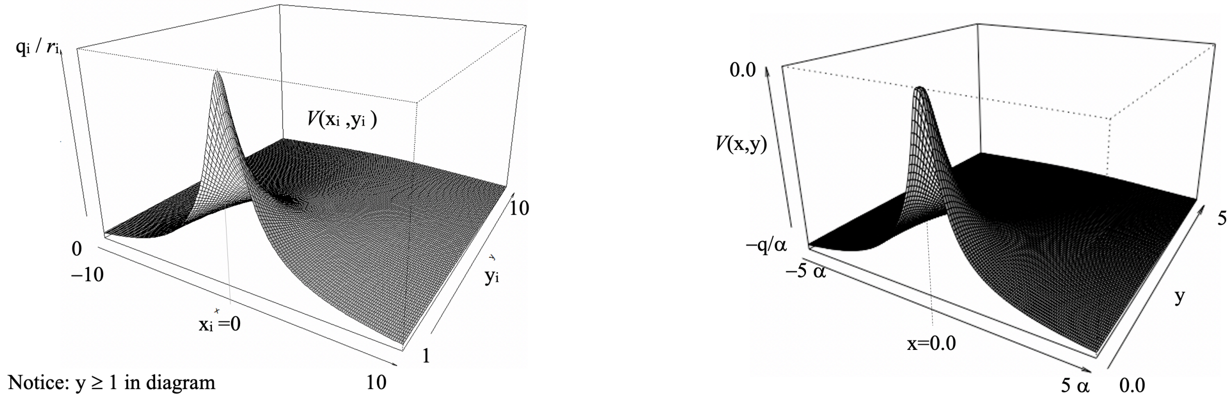
where α is a positive constant with units of distance and q is the total amount of charge included in the symmetric distribution out to an infinite distance. The field strength no longer possesses an illegitimate term; we have only.

$$E(r) = -\frac{1}{\alpha}\frac{\partial}{\partial r} q(r) = q(r)/r^2$$

A major theoretical problem is resolved by incorporating the boundary condition at the origin.

The constant α is a constant of integration

The difference in the form of the force law derives ultimately from the difference in the formulation of the potential, whose only justification is that its gradient yields the experimentally observed force field. The subtle and not so subtle difference in these formulations is illustrated in the following figure.



Potential as conventionally conceived ***Potential (V(r) ~ q(r)) in the current conjecture***

Differently formulated potentials

The uniqueness theorem for the Poisson equation states that for partial boundary conditions, there may be many solutions for the potential, but the gradient of every one of them must be the same so a unique field strength vector derives from *any* potential satisfying the equation. The significance of the solution we found in option 3 is that it satisfies *all* boundary conditions throughout space with finite (in fact, zero charge density and potential) values at the origin and at an infinite remove. The crux of the uniqueness theorem is that if a potential satisfies the Poisson equation as well as meeting boundary conditions throughout the region for which $V(r)$ is defined, then it is the *only* solution. This then is the proof that our conjecture of a solution in option 3 is the correct (and only) solution and the previously accepted inverse square solution must be rejected.

the extent of the implied differences in conjectured potential

It is important to notice how closely the alternative potential field can be characterized as the inverse of the separation of one charge from another, a concomitant of inverse square law forces. The form that has been accepted for centuries (albeit without the possibility of experimental verification at the submicroscopic level applicable to fundamental particles) is shown as the upper curve in the figure below. The figures below provides log plots of the alternative forms of the potential. Even if applied to up and down quarks the new conjecture, that is shown below the traditionally accepted form in the figure becomes virtually indistinguishable at distances greater than 10^{-12} cm. By extending the domain of the plot out to a distance of 10^{-8} cm in the log plot of the next figure, we see that there is virtually no difference at all due to this change in hypothesis.

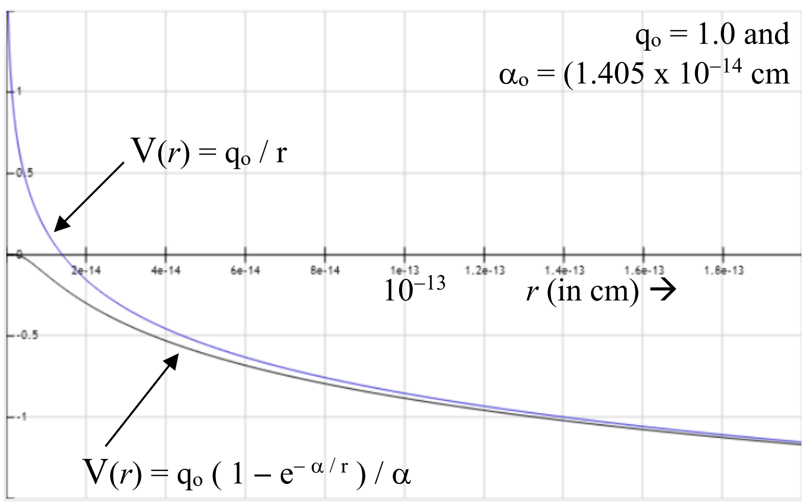


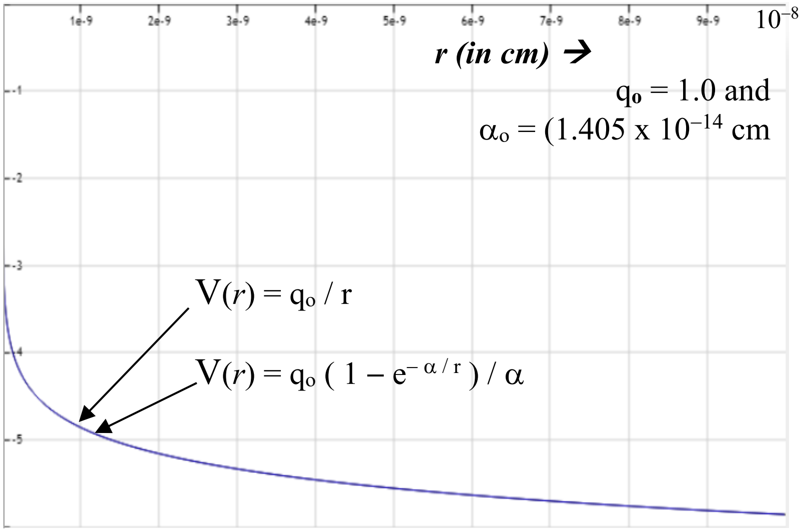
Figure 1.1: Potential as a function of radial distance of a distribution (for r < 2 x 10^{-13} cm)

It is through the polynomial expansion of the exponential that we find agreement with the conventional theory. The series expansion of an exponential is given by the following:

$$e^{-x} = 1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \cdots$$

Thus, we arrive at an approximation for V(r) appropriate whenever r^2 is appreciably greater than α as follows:

$$V(r) = q_o \left\{ + \frac{1/r}{1!} - \frac{\alpha / r^2}{2!} + \frac{\alpha^2 / r^3}{3!} - \frac{\alpha^3 / r^4}{4!} + \cdots \right\}$$



Log of potential as a function of radial distance of a distribution (for r < 1 x 10^{-8} cm)

The significance of the alternative is that it completely avoids a singularity. In this approach both the charge and rest mass embodied in particles can legitimately be associated with continuous distributions of charge and gravitational mass instead. 'Particles' are nonetheless indivisible units. This alternative also effectively eliminates the suspect action-at-a-distance aspect of classical electrostatic and gravitation theories.

Why is this solution to the Poisson equation more legitimate?

It is imperative to understand the details of the derivation of the electric/gravitational potential energy expression from first principles. The Poisson equation is where the discussion must start. It provides a proper formulation of electrostatics and gravitation in a way that can be peeled back one layer at a time to address each theorem that went into its derivation. We do this for a symmetric distribution as follows:

$\nabla^2 V(r) = 4\pi\, \rho(r)$ *vector expression* of *basic Poisson equation*

The left-hand side of this equation is elaborated as follows for symmetric functions.

$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V(r)}{\partial r} \right) = \nabla^2 V(r)$ *symmetric version of left-hand side*

And, of course, from various theorems and experimental results:

$\frac{\partial V(r)}{\partial r} = -\, E(r) = -\, q(r) / r^2$ *symmetric force field as the gradient of a potential field*

So, on the left-hand side we have:

$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V(r)}{\partial r} \right) = \frac{1}{r^2}\frac{\partial}{\partial r}\Big(q(r) \Big)$

On the right-hand side the basic Poisson equation in symmetric situations, there is an obvious inference from spherical symmetry of the charge/mass density in that it is the rate of change with respect to the radius of the total encapsulated symmetric charge/mass divided by the spherical surface area, which follows from the definition of q(r):

$$q(r) = \int_o^r 4\pi\, \rho(r')\, r'^2\, dr' = \int_o^r \frac{\partial q(r')}{\partial r}\, dr'$$

which is tantamount to:

$$\rho(r) = \frac{1}{4\pi r^2} \frac{\partial q(r)}{\partial r}$$

Thus, the symmetric Poisson equation can be transformed into the trivially-satisfied relationship:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(q(r)\right) = \frac{1}{r^2}\frac{\partial q(r)}{\partial r}$$

This is because we rejected a presumption that $V(r) = -\,q(r) / r$, which has previously been the universally accepted expression for potential energy in spherically symmetric situations.

Otherwise we would have had the erroneous solution:

$$\frac{\partial V(r)}{\partial r} = q(r) / r^2 \; - \frac{1}{r}\frac{\partial}{\partial r} q(r)$$

And Poisson's equation would have become:

$$\frac{\partial^2 q(r')}{\partial r^2} = -\frac{1}{r}\frac{\partial\, q(r)}{\partial r}$$

For which we would obtain the following unrealistic expression for charge:

$\ln(q(r)) = -\ln r + C_2 \;\; \Rightarrow q(r) = q / r$

This involves an infinite value at the origin (the essence of a point charge) which defies reasonable boundary conditions.

Thus, it is imperative that we accept instead, the expression:

$$V(r) = (\, q_o - q(r)\,) / \alpha$$

With the experimentally and theoretically acceptable gradient relation,

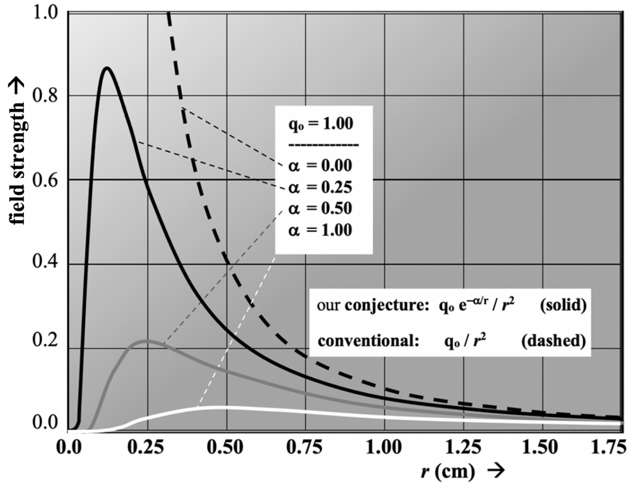
$$\frac{\partial V(r)}{\partial r} = q(r) / r^2$$

We previously introduced the required solution for the charge or mass q(r). Here we list the expressions for all of the associated theoretical constructs.

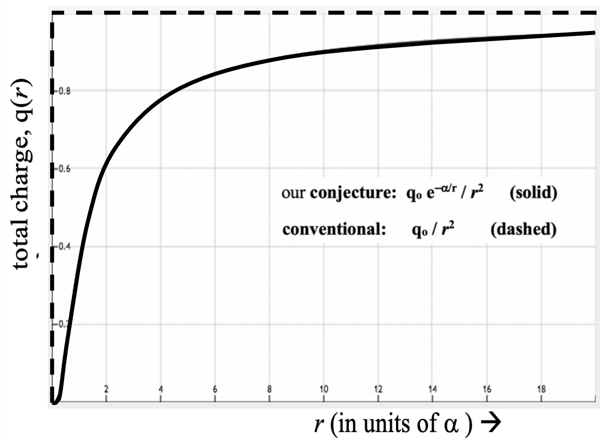
$$\rho(r) = \frac{\alpha\, q_o}{4\pi r^4}\; e^{-\alpha/r} \qquad\qquad q(r) = q_o\, e^{-\alpha/r}$$

$$V(r) = q_o\,(\,1 - e^{-\alpha/r}\,) / \alpha \qquad\qquad E(r) = q_o\, e^{-\alpha/r} / r^2$$

Plots of q(r) and E(r) illustrate the differences in these constructs from the traditional approach; they are equivalent when $\alpha = 0$. With determined values of α there is no experimental difference.

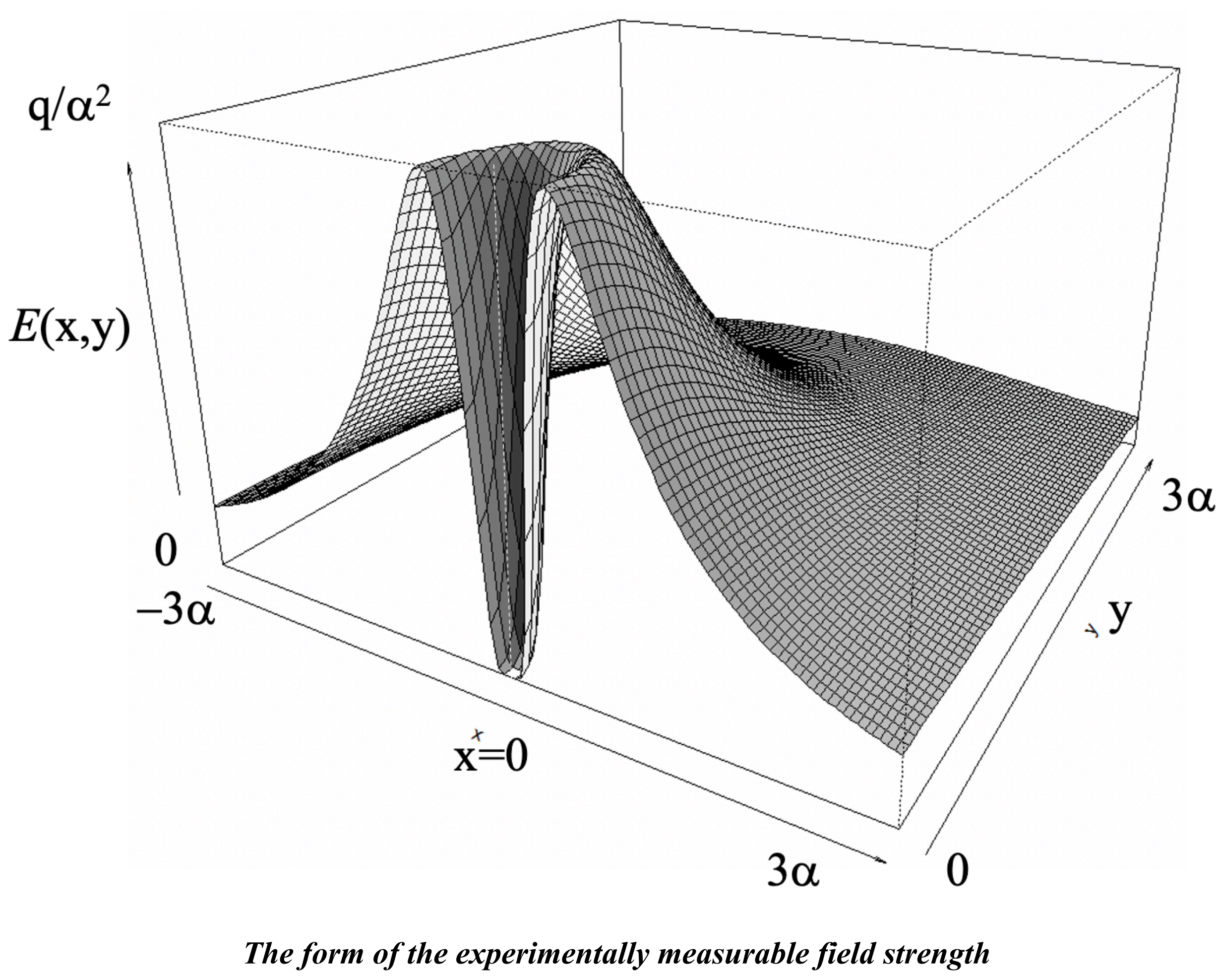


Field strength E(α,r) **– conventional and current view**



Total charge in the current conjecture

The dependence on the variance α of experimentally measurable quantities



The form of the experimentally measurable field strength

The preceding analyses are compatible with all theoretical considerations and experimental results of classical electrostatics and gravitation. It has consistent application to fundamental particles. With the corrected boundary conditions, the gradient in the derivation above can now be taken without compunction. There is no singularity in the charge density at the origin, alleviating the necessity for point particles. The field strength approaches the inverse square law functionality within the accuracy of any possible measurement as the distances from the 'point' at the center of a particle increases. Fundamental particle variances α (in units of centimeters), are less than 10^{-13} cm so that the inverse square dependence certainly applies to the accuracy of experimental possibility for atomic separations greater than 10^{-8} cm.

The implications of this corrected approach to solution of the Poisson boundary value problem are profound. There is now a calculable self-energy associated with substance distribution of the fundamental particles, which translates into distributions that encapsulate both the charge and rest mass of fundamental particles as we will show in another paper on this site. We will also address the coincidence of gravitational as well as electrostatic aspects of fundamental particles. We will then demonstrate the logical development of the hierarchy of fundamental particles.