Self-Energy of Charge Distributions Associated with Fundamental Particles as Their Rest Mass

In the previous paper on this site, we demonstrated that a continuous charge distribution satisfies the Poisson boundary value problem for a fundamental particle of a given charge, q. That distribution is characterized by the amount of charge encapsulated within a spherically symmetric region of radius:

$$q(r) = q\, e^{-\alpha/r}$$

The variance of this distribution is given by α, but what *is* α?

The charge singularity encountered at the origin has precluded a determination of a self-energy of the charge distribution associated with the particle. With an integrable solution to the charge distribution, that determination is now allowed using traditional methods. This involves an assessment of the *work* required to assemble the distribution one infinitesimal piece at a time using an approach with which Newton would have been comfortable.

energetics of charge distributions

Newton defined 'work' as the amount of energy that would have to be expended to move an object through a given distance, $d_2 - d_1$, against a force $F(x)$, $d_1 \le x \le d_2$ as follows:

$$W(d_2 - d_1) = \int_{d_1}^{d_2} F_t(x)\, dx = q_t\,(V(d_2) - V(d_1))$$

The final expression applies to electrostatics where the required expenditure of energy to move a charge from d_1 to d_2 is equal to the amount charge times the difference in potential between the two locations. Integrating the field strength (the force on a unit of charge) along a path from an infinite distance to a distance r determines the work $w(r)$ required to move that unit of charge from that infinite distance to a location a distance r from the center. In the conjecture currently under investigation symmetric charge is located throughout concentric spheres of radii 0 to ∞, and by Gauss's law:

$$w(r) = \int_{\infty}^{r} E(r')\, dr' = q_o \int_{\infty}^{r} (e^{-\alpha/r'}/r'^2)\, dr' = q_o\,(e^{-\alpha/r} - 1)/\alpha\, \Big|_{\infty}^{r} = -V(r)$$

This results in the negative of the potential at location **r** because the force is in the opposite direction of the required motion. Significantly, the integral over the entire distribution of charge defines a self-energy that cannot be accommodated by the singularity central to the traditional approach.

self-energy of a Poisson charge distributions

The distribution implied by our conjecture is not comprised of dispersed point charges, nor yet of *units* of charge, but of a continuous charge density $\rho(r)$. In an increment of volume dv at location **r**, the infinitesimal amount of charge is:

$$\rho(r)\, dv = \frac{\alpha\, q_o}{4\pi\, r^4}\; e^{-\alpha/r}\, dv$$

The incremental amount of energy involved in placing this amount of the total charge into position at location **r** of the distribution is given by the following:

$$W(r)\,dv = q_o^2\,(e^{-\alpha/r} - 1)\,e^{-\alpha/r}\,dv / 4\pi r^4$$

This is the work required to move the infinitesimal amount of charge from an infinite remove, more or less as if constructing the distribution one piece at a time from the center outward as illustrated in figure 1. The result is the positive potential energy of the charge at that location.

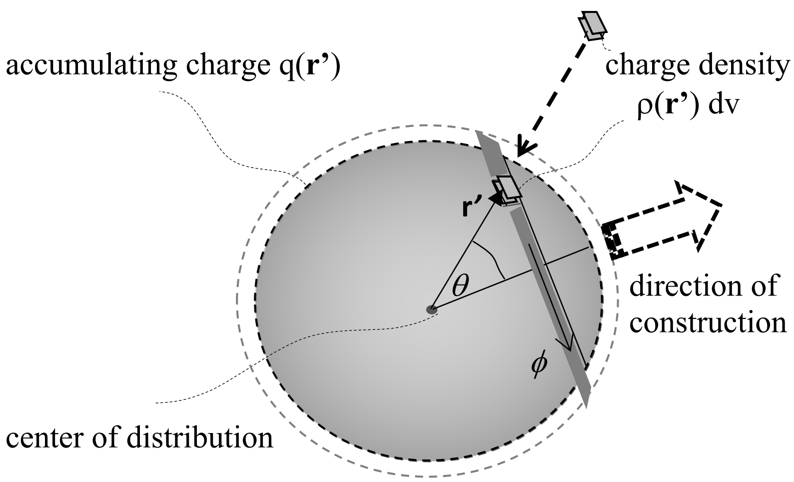

Figure 1: The 'self-energy' of a constructed charge distribution

To obtain the total self-energy S(r) installed in the distribution out to the distance r from the center involves taking the integral of this throughout the entire volume to that distance as follows:

$$S(r) = 4\pi \int_o^r W(r)\, r^2\, dr = q_o^2 \int_o^r [\,(1 - e^{-\alpha/r})\, e^{-\alpha/r} / r^2\,]\, dr$$

$$= q_o^2\,(1 - \tfrac{1}{2}\, e^{-\alpha/r})\, e^{-\alpha/r} / \alpha$$

This terms in this accumulative self-energy are plotted in figure 2. The total 'self-energy' S_{total} contained in the entire distribution out to an infinite distance is:

$$S_{total} = q_o^2 / 2\,\alpha$$

Significantly, we have been able to determine the 'total potential energy' for a Poisson charge distribution. It is straight-forward to associate what we are calling the 'self-energy' of such a distribution with this value. Ultimately, of course, it is reasonable to further associate self-energy of a charge distribution with the mass of a so-represented *particle*.

In this endeavor we thus defer to Einstein's famous mass/energy relation. Setting $S_{total} = m_o c^2$ obtains the following equivalences:

$$m_o = q_o^2 / 2\, c^2\, \alpha$$

$$\alpha = q_o^2 / 2\, m_o\, c^2$$

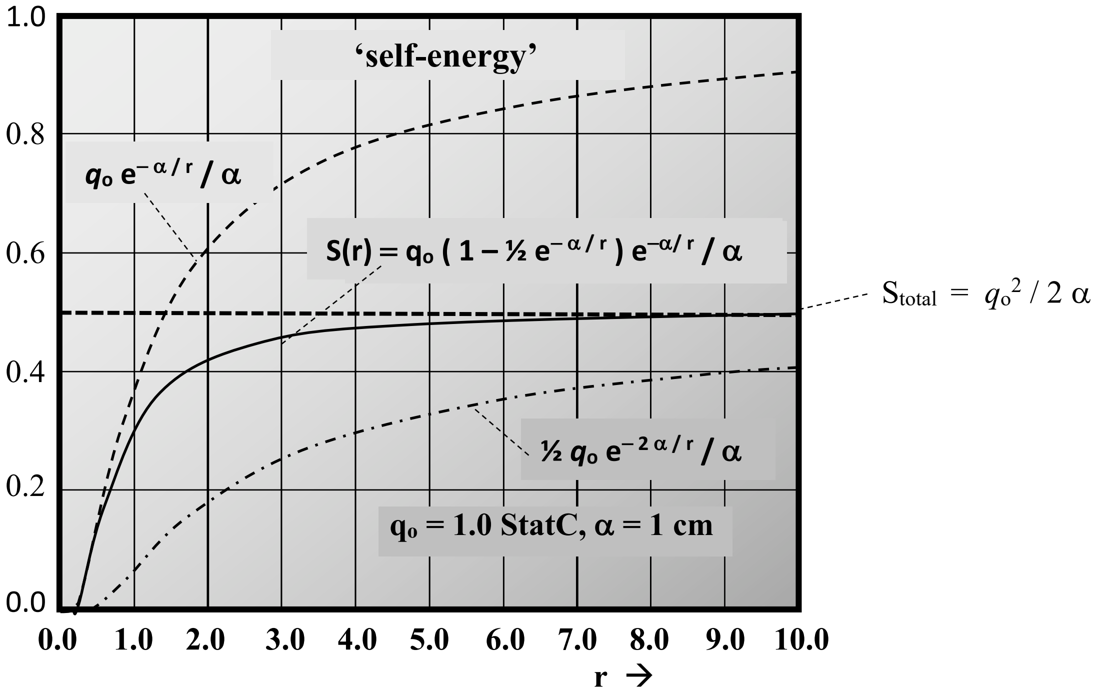

Figure 2: Accumulative 'self-energy' – of an indivisible charge distribution

From these relations it is interesting to explore the implied dimensions of fundamental particles such as the electron whose charge is 4.8032×10^{-10} StatC and mass is 9.11×10^{-28} gm. We obtain as the size dimension, $\alpha_e = 1.409 \times 10^{-13}$ cm. This is to be compared with the 'classical electron radius' of 2.82×10^{-13} cm. The comparison of a 'radius' is not precise in as much as in the Poisson distribution about half of the charge is beyond the radius α_e. For the proton, if we were to assume a single center of charge, which is not justified because of its quark makeup, $\alpha_p = 7.6635 \times 10^{-17}$ cm based on the proton mass of 1.6726×10^{-24} gm.

charge and rest mass as completely defining the particle

Thus both of the constants of integration obtained in the derivation of the charge distribution can ultimately be determined in terms of the physical properties of charge q_o and mass m_o of a particle associated with the distribution. Properties of a basic particle are thus associated with the distribution of charge density as follows:

$$\rho(r) = (q_o^3 / 8\pi\, m_o\, c^2\, r^4)\; e^{-\, q_o^2 / 2\, m_o c^2\, r}$$

Thus the particulars of the charge distribution are completely determined by the total amount of charge and mass of the associated particle. We have found compatibility between a continuous charge distribution and properties of fundamental particles. This provides a fuller understanding of what the term 'particle' really means. The it has previously been associated exclusively with discrete 'chunks' of charge and mass and probably always will be. But whatever the particles that possess the charge and mass, it is the form of the distribution of charge and mass that warrants that designation.

The uniqueness theorem for the Poisson equation states that for partial boundary conditions, there may be many solutions for the potential, but the gradient of every one of them is the same.

So, yes, there is a unique field strength vector derived from *any* potential that satisfies the equation. The significance of the solution we have found is that it satisfies all boundary conditions throughout space with finite (in fact, zero) values at the origin and at an infinite remove. The crux of the uniqueness theorem is that if a potential satisfies the Poisson equation as well as meeting boundary conditions throughout the region for which $V(r)$ is defined, then it is the *only* solution. This then is the proof that our conjecture is the correct solution and the previous traditionally accepted one must be rejected.

The field strength, potential, and charge density each apply at every location in space. So the field strength vector (force) is not determined as an inverse of the distance from a remote charge; that is what wrongly suggested the concept of action-at-a-distance in the first place. That archaic concept not only offended Michael Faraday but even Newton who had proposed it, so the 'added benefit' of this conjecture is that now we can discard the ridiculous concept of action-at-a-distance like animism had been before. In fact the constructs *all* apply at, and pertain *only* to the location specified by $\mathbf{r}$ without restriction; mutually implying each other at that point.