Concerning the Equivalence of Inertial and Gravitational Mass

By virtue of having determined an expression for the self-energy of combined Poisson distributions of electric and gravitational charge, we find ourselves with an oar in the water in a great debate with the pillars of physical science Newton, Eotvos, Einstein, and Dicke. To avoid the issue of singularities at the center of fundamental particles required the acceptance of associating these particles with continuous distributions of charge rather than a mathematical point. Having demonstrated the proper solution to the Poisson boundary value problem, we found that its self-energy could now be calculated in a straight-forward manner, which it could not with the formerly accepted point particle solutions.

$$\mathrm{m_o}\, c^2 = q^2 / 2\, \alpha_e$$

where $\mathrm{m_o}$ is the equivalent mass associated with the electrostatic self-energy, q is the electric charge associated with the particle, and α_e is the variance of the electrostatic charge distribution. The association of this self-energy with the rest mass of the particle determines the value of the variance α_e of the distribution in terms of rest mass and charge.

Since gravitational aspects of particles involve virtually identical theoretical treatment to the electrostatic aspect with electrostatic charge replaced by $\sqrt{G}\, m$, where the italicized *m* is presumably 'gravitational mass'. We find gravitational self-energy must thereby imply a much smaller gravitational component of a total rest mass:

$$m_o\, c^2 = G\, m^2 / 2\, \alpha_m$$

In combining the electrostatic and gravitational aspects of fundamental particles, questions arise regarding a continuing debate concerning the supposed equivalence of inertial and gravitational mass. Equivalence was assumed by Newton after resolving it to one part in a thousand; it remained an open experimental question for Eotvos who increased the resolution considerably; for Einstein precise equivalence became a principle; Dicke and his team proved it to one part in several billion; and more recent measurements assess it at one part in ten to the fifteenth. The mere formulation of the question supposes a difference.

Inertial mass fits into the scheme even when addressing only static situations as we have. Self-energy must associate with inertial mass in static situations where $E = m_o\, c^2/\sqrt{1 - v^2/c^2} \cong m_o\, c^2 + \frac{1}{2}\, m_o\, v^2 + \ldots$ In these expressions the 'rest mass energy' m_o (self-energy) is the energy of a particle when there is no relative velocity. Gravitational mass is, of course, the mass associated with what we have called gravitational charge, $\sqrt{G}\, m$. So the question arises: In our expression for self-energy, is $m = m_o$? What are the implications to this scheme, and does it provide a significant reason to accept or reject the notion of inertial and gravitational mass equivalence?

$$m_o\, c^2 = (q^2/ \alpha_e - G\, m^2/ \alpha_m)/ 2$$

If the mass constructs are equivalent, i.e., if $m = m_o \rightarrow m$, then the preceding equation is a quadratic equation for the mutual m:

$$A\, m^2 + B\, m + C = 0, \text{ where } A = G / 2\alpha_m, B = c^2, \text{ and } C = - q^2 / 2\alpha_e$$

$$m = - B/2A \pm \sqrt{(B^2 - 4AC)/4A^2} = (B/2A) (- 1 + \sqrt{1 - 4AC/B^2})$$

Since $4AC/B^2$ is much less than unity for fundamental particles, we obtain:

$$m \cong (B/2A)\, (2\, AC/B^2) = C/B = q^2 / 2\, c^2\, \alpha_e = m_o$$

Thus, the mutual m is on the order of the rest mass m_o determined exclusively using electrostatic charge. The resolution of this approximation is on the order $16\, [(G/ c^2\, 2\alpha_m)\, (q^2/ c^2\, 2\alpha_e)]^2 < m_o^2$, i.e., less than 10^{-50} grams of the precise value.

If on the other hand, we were to assume that the two types of mass do in fact differ, then we would obtain:

$$m_o = q^2 / 2\, c^2\, \alpha_e + G\, m^2/ 2\, c^2\, \alpha_m$$

Where now we obtain virtually the same value for m_o with a difference in the second term of on the order of 10^{-70} grams. Lest we rush to conclude that this is a difference of no consequence, we must consider as did Robert Dicke, the situation of the annihilation of matter and anti-matter particles. He considered that whether the particle masses are the same or different, since there is no levitating negative mass to cancel gravitational mass, once the electrostatic charges have been canceled by annihilation, what happens to the associated gravitational charges, i.e. gravitational mass? It cannot be canceled and must, therefore, be associated with a residual self-energy as follows:

$$m_o' = G\, m^2 / 2\, c^2\, \alpha_m$$

This truly diminutive residual term is on the order of 10^{-70} grams.

But what do we make of that?

The resolution of this issue of supposed non-annihilating mass is addressed on this site in the article, Poisson Distribution Interactions: Fragmentation, Annihilation, and Indivisibility.