Poisson Distribution Interactions: Fragmentation, Merging, Annihilation, and Indivisibility

Fundamental particles are the basis of the atomic theory of matter; indivisibility must pertain at the most fundamental level. The introduction of the concept of fundamental particles consisting of distributed rather than point charges cannot change that requirement. Of course there is gravity that must involve Poisson distributed gravitational mass to be dealt with – and rest (is that inertial?) mass as well. We will deal with the gravitational aspect in this paper, since fundamental particles possess both electric charge and gravitational mass, how these two distributions fit into the same particle is a significant issue that must be addressed, at least summarily here. In the point particle treatment, is a mass located at the very same 'point' as the electric 'charge'? In the quantum probabilistic distribution of the electron about the nucleus of an atom, is that distribution the same for the electronic charge as for the electron's mass? If the answers to these questions are 'yes' in both instances as one must suppose, is a separate force involved in keeping them coincident? How do electric and gravitational fields associate? For Poisson distributions of charge and gravitational mass, these questions are secondary to the more general question of how Poisson distributed substances interact.

The nature of a fundamental particle involves an associated electrostatic Poisson distribution; it provides the direct connection of the given amount of electric charge and self-energy (mass) that fully determine the form of the distribution. But how is that self-energy related to the binding and repulsive energy associated with interactions with other particles (distributions)?

How do Poisson distributions interact?

We assume that charge density and associated fields at each location in space are additive without altering the distribution associated with each particle. So midway between identical particles half of charge is associated with each particle. But the force of interaction is the product of the charge density of one multiplied by the field strength of the other integrated over all space. The total force is the sum of the integrals of the effect of each one on each of the others. Similarly the energy required to hold two particles at a given distance is the sum of the potential energy of the charge density of each one in the electric field of the other. See the illustration of figure 1.

For two distributions whose centers are separated by the distance R, the integrals for determining the total energy $\mathcal{E}$ of a two-particle Poisson distribution can be expressed as follows:

$$\mathcal{E}(\mathrm{R}) = 4\,\pi \int_0^\infty [\,\mathrm{V}_1(\mathbf{r}_1) + (\,\mathrm{V}_2(\mathbf{r}_2)]\,[\rho_1(\mathbf{r}_1) + \rho_2(\mathbf{r}_2)]\; r'^2\, d\mathbf{r}'$$

Here V is potential, and ρ is charge density. We can use $d\mathbf{r}' = d\mathbf{r}_1$, where $\mathbf{r}_1 = \mathbf{r}_2 + \mathbf{R} \rightarrow d\mathbf{r}_1 = d(\,\mathbf{r}_2 + \mathbf{R}\,)$ or we can use $d\mathbf{r}' = d\mathbf{r}_2$ where $\mathbf{r}_2 = \mathbf{r}_1 - \mathbf{R} \rightarrow d\mathbf{r}_2 = d(\,\mathbf{r}_1 - \mathbf{R}\,)$. There are four integrals implied by the expression of the integrand that we denominate by associating indices as follows:

$$\mathcal{E}(\mathrm{R})_{\mathrm{TOTAL}} = \mathrm{S}_{11} + \mathrm{W}_{12}(\mathrm{R}) + \mathrm{W}_{21}(\mathrm{R}) + \mathrm{S}_{22}$$

Two of these terms we refer to as self-energy potentials S; the remaining two terms describe the potential energy W of one distribution with respect to the other, i.e., the energy involved in the binding or repulsion of the two distributions. The self-energy potentials are the following:

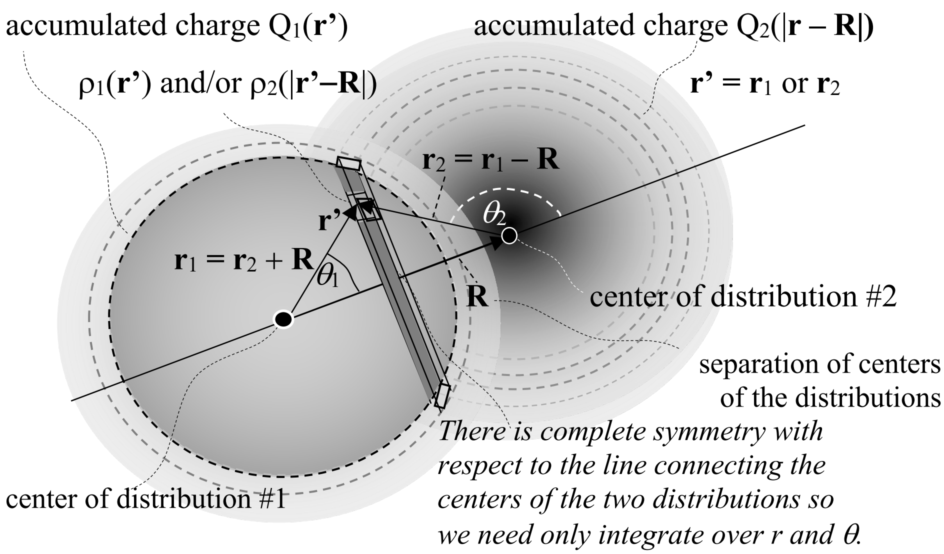


Figure 1: Parameters for determining potential energy of two separate Poisson distributions

$$S_{11} \;=\; 4\,\pi \int_o^{\infty} V_1(r_1)\;\rho_1(r_1)\;r_1^2\;d\,r_1$$

$$S_{22} \;=\; 4\,\pi \int_o^{\infty} V_2(r_2)\;\rho_2(r_2)\;r_2^2\;d\,r_2$$

These terms are independent of the extent of the separation of the distributions even though R does appear in the integrand of S_{22}. Per discussions elsewhere on this site, $V_1(r_1) = q_1\,(1 - e^{-\alpha 1\,/\,r1})\,/\,\alpha_1$ and $\rho_1(r_1) = q_1\,\alpha_1\,e^{-\alpha 1\,/\,r1}\,/\,4\,\pi\,r_1^4$. Both integrands involve only the respective total charge and charge variance, i.e., q_1 and α_1 as well as q_2 and α_2, each pair is associated with a *single* distribution independent of where the center of that individual distribution is located in space.

$$S_{11} = 4\,\pi \int_o^{\infty} [\,(q_1\,(1 - e^{-\alpha 1\,/\,r1}\,)\,/\,\alpha_1\,)\,(q_1\,\alpha_1\,e^{-\alpha 1\,/\,r1}\,/\,4\,\pi\,r_1^4\,)\,]\,r_1^2\;d\,r_1$$

$$= \; q_1^2 \int_o^{\infty} [\,e^{-\alpha 1\,/\,r1}\,(\,1 - e^{-\alpha 1\,/\,r1}\,)\,/\,r_1^2\,]\;dr_1 \;=\; q_1^2\,/\,2\,\alpha_1$$

Similarly, since $r_2 = r_1 - R$ and $dr_2 = d(\,r_1 - R\,)$, we use the same formulation to obtain:

$$S_{22} = 4\,\pi \int_o^{\infty} [\,(q_2\,(1 - e^{-\alpha 2\,/\,r2}\,)\,/\,\alpha_2\,)\,(q_2\,\alpha_2\,e^{-\alpha 2\,/\,r2}\,/\,4\,\pi\,r_2^4\,)\,]\,r_2^2\;d\,r_2$$

$$= \; q_2^2 \int_o^{\infty} [\,e^{-\alpha 2\,/\,r2}\,(\,1 - e^{-\alpha 2\,/\,r2}\,)\,/\,r_2^2\,]\;dr_2 \;=\; q_2^2\,/\,2\,\alpha_2$$

These two self-energy terms, one for each interacting particle, are the same no matter where the distributions are located.

There are also the two cross terms determining the relative potential energy of one distribution with respect to the other based upon the position **R** of its center relative to that of the other. The first of these cross terms is the following:

$$W_{12}(R) = \int_0^\infty [\, (q_1 (1 - e^{-\alpha 1 / r'}) / \alpha_1) (q_2\, \alpha_2\, e^{-\alpha 2 / (|r' - R|)} / (|r' - R|)^4)) \, r'^2 \, dr'$$

Integration parameters in $W_{21}(R)$ can be manipulated to demonstrate the same result as $W_{12}(R)$ but with indices reversed. We must sum those two terms to obtain the total repulsive (or binding) energy. In either case we must integrate the effects of the density of the second distribution at each point in space with respect to the potential of the first out to that point.

We are used to the potential energy of an apple before it falls from a tree being determined as the product of the mass of the apple times the gravitational potential of the earth – a single term. So why are there two terms being used here? The better question, and one that is easy to answer, is why is only one term employed with regard to the apple and the earth? The reason is that the force on the apple involves the acceleration of the apple caused by the mass of the earth; adding a term involving the acceleration of the earth caused by the mass of the apple is ridiculous. Falling objects are what are used to measure the gravitational constant. Like so much in scientific formulations, higher order terms and insignificantly small terms are lopped off of useful formulas.

To summarize: the potential energy of two indivisible distributions is the sum of potential energies $V(\mathbf{r})$ of each potential field with respect to the sum of the distributed charge densities $\rho(\mathbf{r})$ of both the distributions. This is expressed by the following equation:

$$W(R) = \int_0^\infty [\, V_1(r) + V_2(|\mathbf{r} - \mathbf{R}|)\,]\, [\rho_1(\mathbf{r}) + \rho_2(|\mathbf{r} - \mathbf{R}|)\,]\, \mathbf{dr}^3$$

$$\mathbf{dr}^3 = r^2 \sin\theta \, d\phi \, d\theta \, dr = 2\pi \, r^2 \sin\theta \, d\theta \, dr$$

$$|\, \mathbf{r} - \mathbf{R} \,| = R \sqrt{1 - 2\, (r_1 / R)\cos\theta_1 + (r_1/R)^2}$$

As was shown in figure 1, it can similarly be expressed as:

$$\mathbf{dr}^3 = r_2^2 \sin\theta_2 \, d\phi_2 \, d\theta_2 \, dr_2 = 2\pi \, r_2^2 \sin\theta_2 \, d\theta_2 \, dr_2$$

$$|\, \mathbf{r}_2 + \mathbf{R} \,| = R \sqrt{1 + 2\, (r_2 / R)\cos\theta_2 + (r_2/R)^2}$$

merging, fragmentation, and annihilation of electrostatic Poisson distributions

Significantly, if two Poisson distributions, the magnitude of whose total charges and variance are equal, were to interact, then the magnitude of the sum of the interaction terms W will either equal the sum of the two self-energy terms S (or zero if the signs of their charges are opposite) when their separation is zero. This is illustrated in the diagram of figure 2, where underlined parameters pertain to the case where the signs of the charges are opposite.

Parameters that are not underlined in the diagram of figure 2 illustrate the energetics of two completely identical Poisson distributions. The energy required to force two such identical distributions to superimpose themselves upon one another is equal to four times the self-energy of one distribution; this merger would then be associated with a particle (a clump of charge) of twice the electric charge and four times the self-energy. In general, superimposing N identical Poisson distributions could be associated with a different particle with N times the charge and N^2 times the rest mass of the original particles. I say 'might be associate with a different particle' because there is little reason to assume that the distribution would not seek a lower energy configuration of N original particles speeding away from each other. And for that matter, why would the original particle not be disseminated into an infinite number of infinitesimal Poisson distributions? That is the conundrum of fragmentation in which energy is dissipated by separation of a more energetic 'particle' into many lower energy 'particles' while conserving the amount of electric charge. Thus, the term 'particle' as suggesting a Poisson electric charge distribution must be qualified since it does not yet possess the indivisibility of a fundamental particle – a building block of matter.

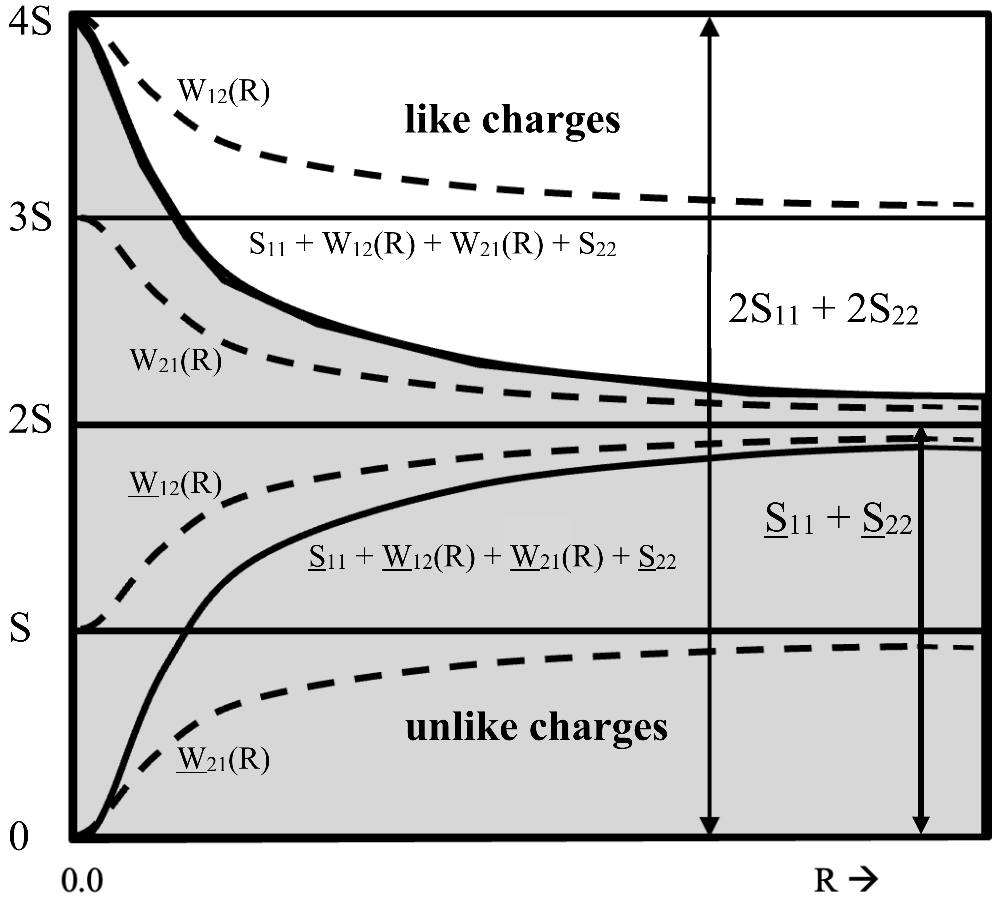


Figure 2: Composite electrostatic Poisson distribution energy components

In a somewhat similar way, two Poisson charge distributions of opposing sign but otherwise identical would seem inevitably to annihilate each other if not forcibly separated, releasing their total rest energy in the process. Annihilation does occur in nature, as for example electrons and beta particles, but the interconnectivity of electric charge and gravitational mass during such a demise demands additional consideration.

That an amount of electric charge and self-energy in free space would distribute itself as a Poisson distribution is unquestionable, but how is an apparent inevitability of fragmentation into increasing numbers of lesser distributions avoided? First, we must elucidate the electrostatic effects.

interactions of the electrostatic Poisson distributions of fundamental particles

Fundamental particles of charge and mass (self-energy) that actually exist are for all intents and purposes indivisible. In pursuing the atomic theory of matter to a level at which indivisibility pertains to particles, up and down quarks are the most likely suspects to satisfy that requirement. The up quark possesses +2/3 of an electronic charge $e = 4.8 \times 10^{-10}$ stat-coulombs. The down quark possesses −1/3 of the electronic charge. Protons and neutrons are known to be comprised of these two more fundamental particles, and the electron while indivisible, may well be equivalently comprised of three down quarks with a total electric charge of $-e$. The proton is comprised of two up quarks and one down quark giving it an electric charge of $+e$. The neutron is traditionally presumed to be comprised of two down quarks and one up quark giving it no net charge. It is a tandem of this structure that exhibit the instability nominally attributed to a neutron. The mass properties I assign to the quarks as fundamental particles have been determined from the known properties associated with the subatomic particles, and thus the self-energies $S_{uu} = 0.0007828$ ergs and $S_{dd} = 9.0967 \times 10^{-8}$ ergs. These properties and the thermonuclear nucleosynthesis involved in the constructive creation of subatomic particles is provided in an article elsewhere on this site.

What is provided here is a determination of energies and forces involved in the interaction of such distributions using equations identified above with numerical integration implemented as ‘R’ programs. The program below performs calculations to plot the repulsive energy of two similar quarks, W_{12} or W_{21} as illustrated schematically in figure 2. Plots of W(R) for two up quarks are shown in figure 3 out to different separation ranges. Figure 4 shows the similar plots for the down quark using the same ‘R’ program but with the down quark values qd replacing qu and ad replacing au in the Cuu integrand calculation, with plot ranges appropriate to the different variance.

```
ER_list_handler <- function(R) {
  return (lapply(R, ER)) }
ER <- function(R) {
  pi<-3.145926
  el <- 4.8e-10
  qu <- 2*el/3
  qd <- -el/3
  au <- 6.53999e-17
  ad <- 1.4071e-13
  Drd <- 1e-19
  Dtheta <- 0.01
  dv <- Dtheta*Drd/2
  E <- 0
  for ( increment in c( 0:( 314.5926) ) ) {
    theta <- increment*Dtheta
    ST <- sin(theta)
    CT <- cos(theta)
    TwoCT <- 2*CT
    for ( r_increment in c( 1:(30000) ) ) {
      r <- r_increment*Drd
      rminR1 <- sqrt(R^2-TwoCT*r*R+r^2)
      Cuu <- qu*qu*(au/au)*ST*(1-exp(-au/r))*exp(-au/rminR1)*dv*r^2/rminR1^4
      Cuu <- qu*qu*(au/au)*(1-exp(-au/rminR1))*exp(-au/r)*ST*dv/r^2
    }  }
  return (E)
}
plot (ER_list_handler, 1e-18, 1e-16)
```

either/or

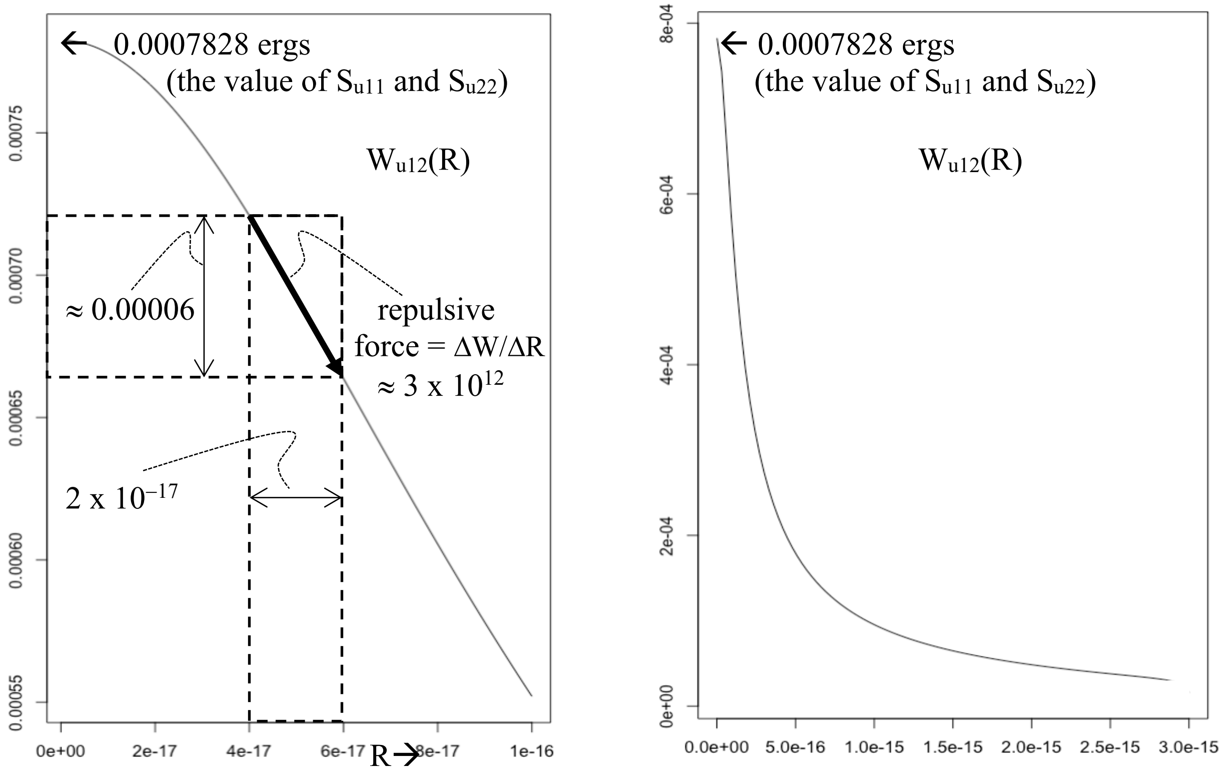


Figure 3: Repulsive electrostatic energy of two Poisson distributions for up quarks

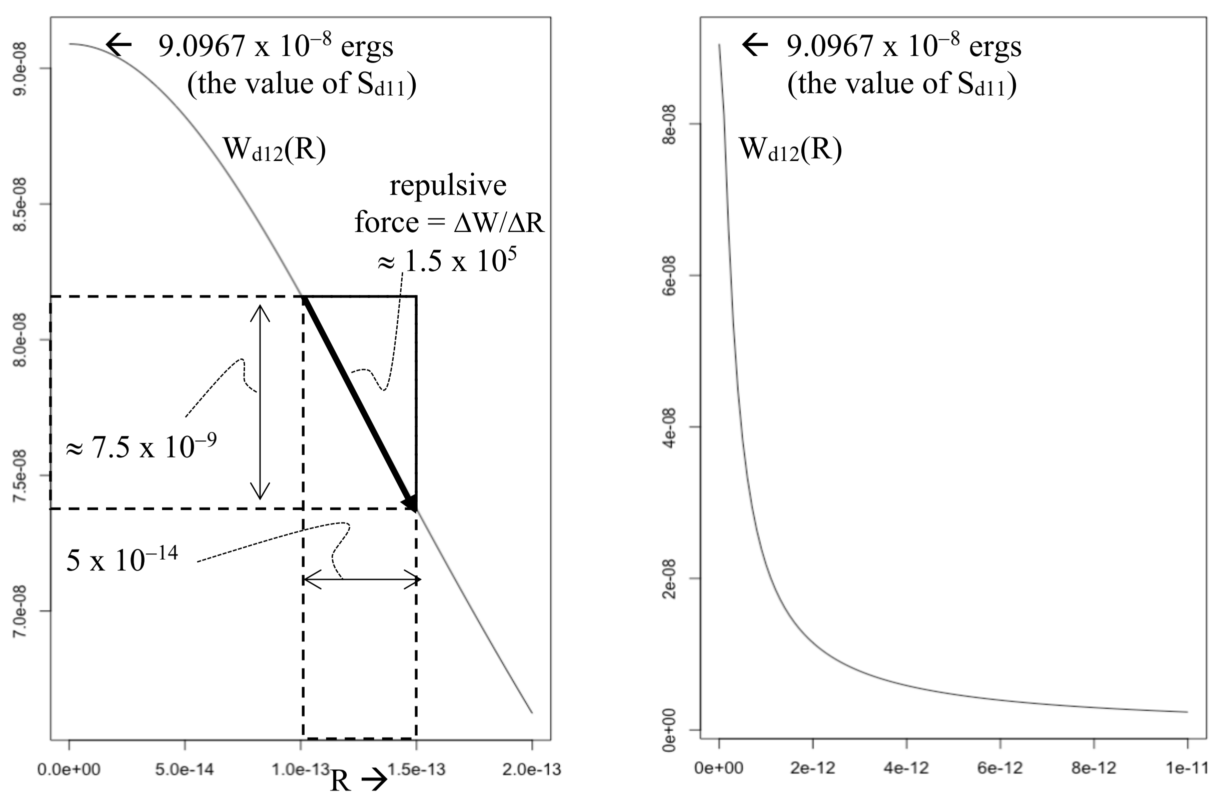


Figure 4: Repulsive electrostatic energy of two Poisson distributions for down quarks

The force F(R) between electrostatic Poisson distributions is obtained similarly by integrating the field strength E(r) times the amount of charge density at each point in space as follows:

$$F(R) = \int_0^{\infty} [\, E_1(r) + E_2(|\mathbf{r} - \mathbf{R}|) \,] \, [\rho_1(\mathbf{r}) + \rho_2(|\mathbf{r} - \mathbf{R}|) \,] \, \mathbf{dr}^3$$

where $E(r) = q(r)/r^2$, with $q(r) = q\,(1 - e^{-\alpha / r})$ the accumulated charge in an electrostatic Poisson distribution out to the distance r from the center. Plots of the force F(R) between two up quarks (left panel) and down quarks (right panel) are shown in figure 5, each for two different separation ranges. These plots were created with the same 'R' program structure but with the expression for the potential Cuu replaced by the expression for force Fuu in the statement of the integrand calculation of the 'R' program shown above. These statements are the following for the associated up and down quarks:

```
Up:     Fuu <- qu*qu*au*CT*ST*exp(-au/rminR1)*exp(-au/r)*dv/(r^2*rminR1^4)

Down: Fdd <- qd*qd*ad*CT*ST*exp(-ad/rminR1)*exp(-ad/r)*dv/(r^2*rminR1^4)
```

The factor CT – cos (Theta) – restricts force components to only those along the direction of the centerline between distributions. The size of dr and number of increments in r

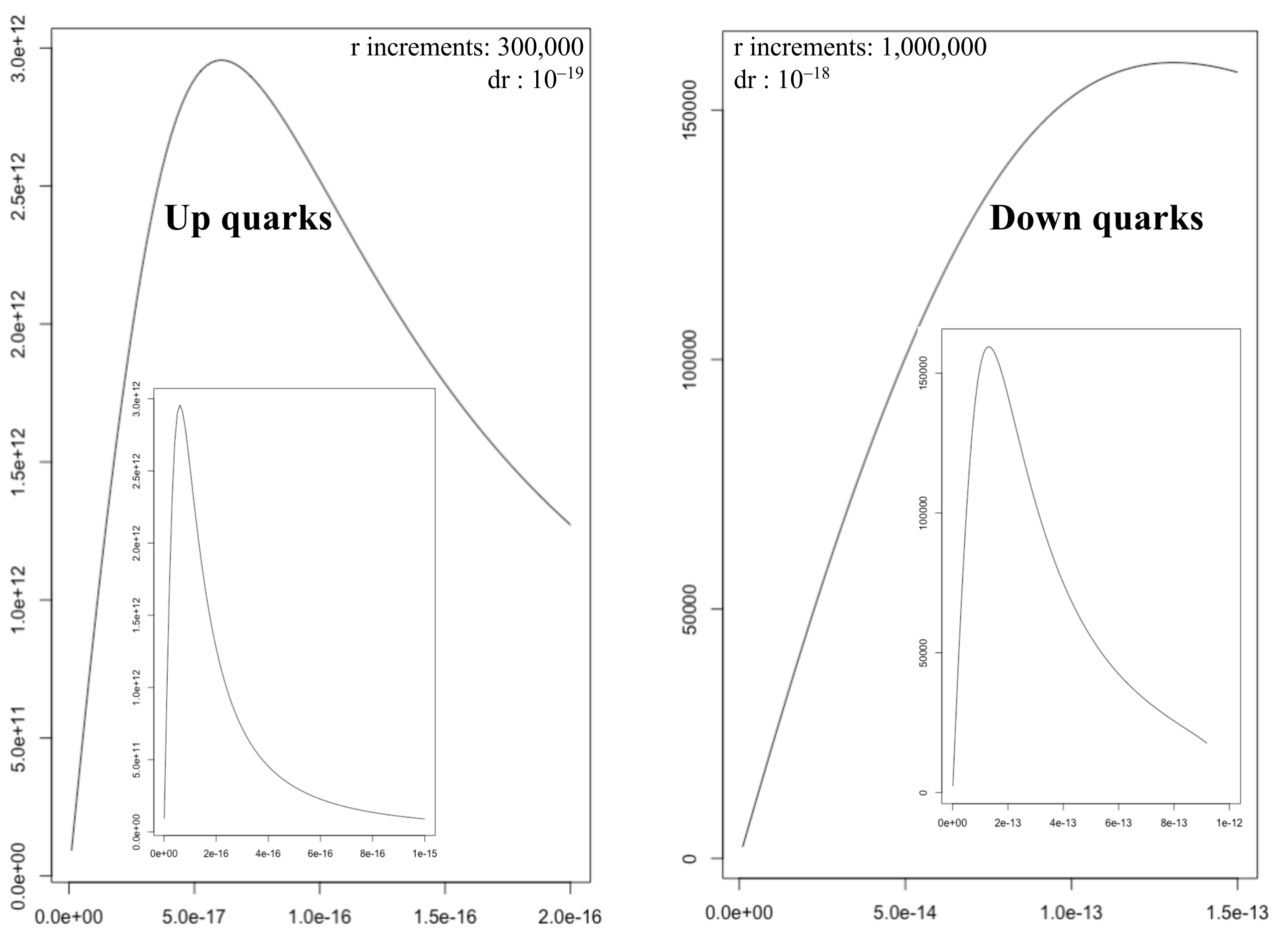


Figure 5: Forces between electrostatic Poisson distributions of two up, and two down, quarks

the role of gravitational Poisson distributions in fundamental particles

Since a Poisson equation applies to gravitation of the same particle for which the electrostatic Poisson distribution applies, how should gravitational mass fits into this scheme? At appreciable distances from the centers of particles gravity is an orders of magnitude weaker force than the electrostatic force, so how could it possibly provide the necessary cohesive strength necessary for indivisibility?

The Poisson distribution is the only viable distribution of charge or mass throughout space that implies precise quantities of the conserved properties of charge (however defined) and energy with reasonable boundary conditions at the origin and at infinity. But the discussion so far has focused on the electrostatic and not the gravitational aspects of a particle. We have attributed electrostatic self-energy to rest mass; do we also assume that this mass is 'gravitational mass' associated with a Poisson distribution with gravitational fields of a similar form to that of the electrostatic fields?

Experiments and observed gravitational effects provide the necessary background for any theoretical approach. Classical gravitation theory introduced the gravitational constant that results in commensurability of mass and electric charge units in force equations. To the same effect, we define what we will call a 'gravitational charge' as $i \sqrt{G}\, m$, where i is the *imaginary* root of minus one and m is the rest mass associated with the self-energy of the electrostatic Poisson distribution. Since both the electrostatic and gravitational Poisson solutions apply to the same fundamental particles, we re-define 'total charge' as $(q(r) \pm i \sqrt{G}\, m(\mathrm{r}))$ and proceed to the complete solution of the force field $\mathrm{F}(r)$ for the total Poisson charge distribution applicable to a fundamental particle.

$$\mathrm{F}(r)\, \mathrm{d}r = \{ [q(r) \pm i \sqrt{G}\, m(r)]\, [\rho_e(r) \pm i \sqrt{G}\, \rho_m(r)]^* / r^2 \}\, \mathrm{d}r$$

Here x^* is the *complex conjugate* of x, such that if $x = a + i\, b$, then $x^* = a - i\, b$. The total force of a particle on another when they are separated by a distance R that is large with respect to a is:

$$\mathrm{F(R)} = q^2/\mathrm{R}^2 - \mathrm{G}\, m^2/\mathrm{R}^2$$

Although these forces are oppositely directed, notice that q^2 is 40 orders of magnitude larger than $\mathrm{G}\, m^2$ for any realized fundamental particle. The rationale for the ambiguous '±' sign of the imaginary factor is that it accommodates correspondence to the sign of the associated electrostatic charge in support of complete annihilation of particles with their anti-particles. We have not yet assigned a value to α_m; a relatively small value accommodates this gravitational aspect of charge near the center of a particle providing the necessary indivisibility of fundamental particles.

We are now dealing with two, at least somewhat separate Poisson distributions associated with the same particle. There are both potential and density parameters associated with the separately accounted effects of the electrostatic and gravitational charge distributions. There is a bifurcation in the integrand of each equation above to incorporate the gravitational aspect in both the self and cross-energy integrals. These remain similar to the formulas introduced above. The adjustment to self-energy is minimal when we include gravitational charge $i \sqrt{G}\, m$ rather than just electrostatic charge q_e. In this case there is no involvement of separation of centers, R to account for gravitational charge because both the electrostatic and gravitational charges must be distributed symmetrically with regard to the very same center of the particle.

$$\mathcal{E}_{\mathrm{total\_e/g}} = \int_0^{\infty} [\, q_e\, (1 - e^{-\alpha_e/r}) / \alpha_e - i \sqrt{G}\, m\, (1 - e^{-\alpha_m/r}) / \alpha_m]\, [\, q_e\, \alpha_e\, e^{-\alpha_e/r} - i \sqrt{G}\, m\, \alpha_m\, e^{-\alpha_m/r}\,] / r^2\, \mathrm{d}r$$

Thus, we arrive at four energy terms similar to electrostatic interactions analyzed above:

$$\mathcal{E}_{\text{total\_e/g}} = S_{ee} + C_{eg} + C_{ge} + S_{gg}$$

$$S_{ee} = q_e^2 \int_0^\infty [(1 - e^{-\alpha_e/r})\, e^{-\alpha_e/r} / r^2]\, dr = q_e^2 / 2\, \alpha_e$$

$$C_{eg} = +\, i\, (\sqrt{G}\, m\, q_e\, \alpha_m / \alpha_e) \int_0^\infty [(1 - e^{-\alpha_e/r})\, e^{-\alpha_m/r} / r^2]\, dr$$

$$= +\, i\, \sqrt{G}\, m\, q_e / \alpha_e \; - i\, \sqrt{G}\, m\, q_e\, \alpha_m / \alpha_e\, (\alpha_e + \alpha_m)$$

$$= +\, i\, \sqrt{G}\, m\, q_e / \alpha_e\, [1 - \alpha_m / (\alpha_e + \alpha_m)] = +\, i\, \sqrt{G}\, m\, q_e / (\alpha_e + \alpha_m)$$

$$C_{ge} = -\, i\, (\sqrt{G}\, m\, q_e\, \alpha_c / \alpha_m) \int_0^\infty [(1 - e^{-\alpha_m/r})\, e^{-\alpha_e/r} / r^2]\, dr$$

CANCELATION
(of imaginary components)

$$= -\, i\, \sqrt{G}\, m\, q_e / \alpha_m \; + i\, \sqrt{G}\, m\, q_e\, \alpha_c / \alpha_m\, (\alpha_e + \alpha_m)$$

$$= -\, i\, \sqrt{G}\, m\, q_e / \alpha_m\, [1 - \alpha_e / (\alpha_e + \alpha_m)] = -\, i\, \sqrt{G}\, m\, q_e / (\alpha_e + \alpha_m)$$

$$S_{gg} = -\, G\, m^2 \int_0^\infty [(1 - e^{-\alpha_m/r})\, e^{-\alpha_m/r} / r^2]\, dr = -\, G\, m^2 / 2\, \alpha_m$$

Of note is the fact that the two imaginary components cancel each other since $C_{ge} = -\, C_{eg}$. Thus, with incorporation of gravity, total energy remains a real quantity comprised of only the difference in the self-energies of the electrostatic and gravitational Poisson distributions.

$$\mathcal{E}_{\text{total\_e/g}} = q_e^2 / 2\, \alpha_e - G\, m^2 / 2\, \alpha_m$$

Clearly, the contribution to total energy of a particle from the gravitational charge is minimal. So, it is also significant that the gravitational self-energy component does not increase or diminish the inertial mass of the particle to any measurable extent. However, if the gravitational variance parameter were sufficiently small, the total energy of the particle could be reduced to zero. In which case the baryonic mass of the universe would be similarly reduced. This issue will be discussed elsewhere in another paper provided on this site.

indivisibility and annihilation of Poisson distributions

The gravitational self-energy term is negative with a relatively appreciable (in comparison to the electrostatic energy) value only in the immediate vicinity of the center of the particle, providing a lego-like latch to avoid fragmentation. Figure 6 is a diagrammatic illustration of the field strength of the total distribution, showing that distribution stability introduced by the gravitational component. Whereas two electrostatic Poisson distributions are bound together, or repelled, by their cross-energy terms, these two united but disparate distributions are part and parcel of the same fundamental particle with a common center. With realistic values of α_m relative to α_e, the separation of appreciably positive (electrostatic) force from the negative (gravitational) force is many orders of magnitude as shown for the field strength of conjoined distributions in figure 7.

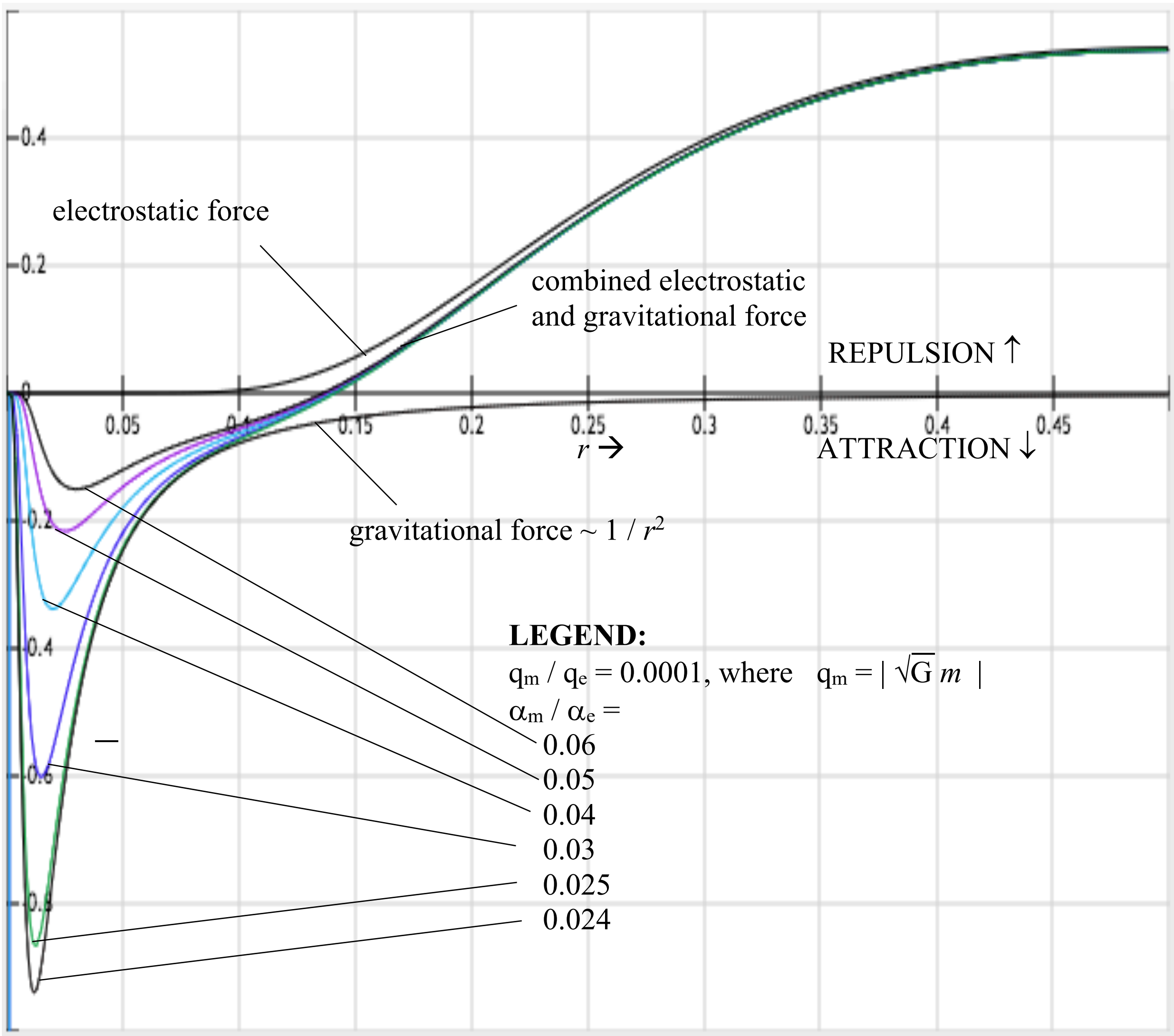
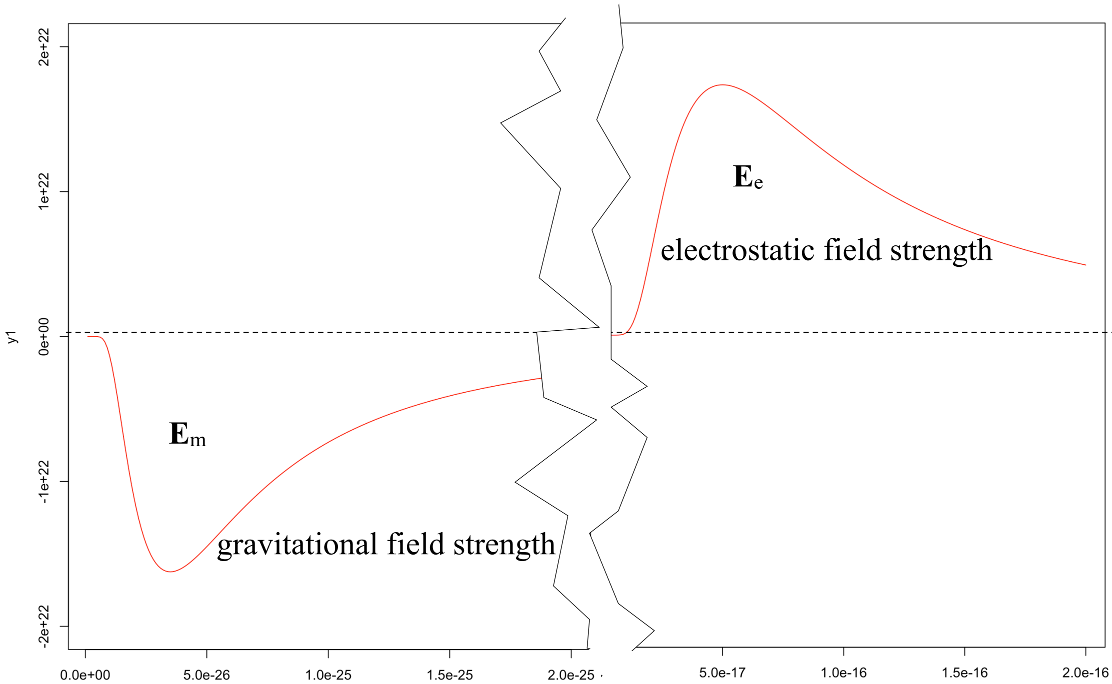


Figure 6: Combined electrostatic and gravitational forces

Employing complex values for the charge in field equations introduces an additional problem for field theory: Fields involve the concept of a ‘unit charge’, so what is a unit complex charge? We assume that the complex unit of charge must be Poisson distributed since it too must satisfy a Poisson equation. The electrostatic component of the unit complex charge we define as q_u. Then the gravitational component must be: $q_{gu} = i\sqrt{G}\, q_{eu}^2 / (2\, c^2\, \alpha_{eu})$. To qualify as a ‘unit’, we must have that:

$$q_u \times q_u^* \equiv q_u^2 = [q_{eu} + i\sqrt{G}\, q_{eu}^2 /(2\, c^2\, \alpha_{eu})]^2 = 1 \rightarrow X - bX^2 = 1$$

where $X = q_{eu}^2$ and $b = G / (2\, c^2\, \alpha_{eu})^2$



R = seq(1e-27,2e-25, 1e-27) R = seq(1e-18,2e-16, 1e-18)

two separate ranges increment sizes for the same function

'R' plot program

```
G = 6.67428*10^(-8)
c = 3*10^10
q = 3.2*10^(-10)
aq = 10^(-16)
am = 7*10^(-26)
m = q^2 /(2*aq*c^2)
print(m) ;                    5.688889e-25
Eq = q*exp(-aq/R)/R^2
Em = -G^0.5*m*exp(-am/R)/R^2
y = Eq+Em
plot(R, y, ylim=c(-2*10^22,2*10^22), type="l", col="red")
```

Figure 7: Exclusivity of electrostatic and gravitational field strength intensity

We can solve the quadratic equation: $X^2 - (1/b)\,X + (1/b) = 0$ to obtain:

$X = (1/2b) - \sqrt{[(1/4b^2) - (4/b)]} = (1/2b)[1 - \sqrt{(1 - 4\,b\,)}] \cong 1$, assumes $b \ll 1.0$, i.e., $\alpha_{eu} \gg 10^{-25}$.

Thus we obtain $q_{eu} \cong 1$ and $q_{gu} \cong \sqrt{G} / (2\,c^2\,\alpha_{eu}) = 1.437 \times 10^{-25}/\,\alpha_{eu}$. So the plot provided as figure 7 provides a realistic approximation of the indivisibility provision of up quark of fundamental particles.

We noted above that the ambiguous '±' sign associated with gravitational charge addresses an issue that Robert Dicke had raised with regard to the debate of whether there is a difference between *inertial* and *gravitational* mass discussed in the article on this site, Equivalence of Inertial and Gravitational Mass. We noted that by virtue of there even being a meaningful question, there must in fact be a meaningful distinction. Dicke opined that when two particles annihilate to eliminate their charge, the question of what happens to their mass is left hanging since, as he pointed out, there is no levitating negative mass. The rationale for the ambiguous '±' sign of the imaginary factor i is that it accommodates correspondence to the sign of the associated electrostatic charge. It provides $(+i^2)$ times the gravitational component of the self-energy for both the particles and $(-i^2)$ times the binding energy of the positive and negative particles so that the cross terms precisely cancel the gravitational interaction aspects just as do the electrostatic aspects. This in turn supports the complete annihilation of particles with their anti-particles – including their mass, with no residual that was of concern to Dicke. So although there is no negative levitating mass, there is a positive and a negative imaginary gravitational charge that can be totally annihilated.