Transaction Theory of Radiation

In this presentation of Maxwell's equations and their solutions, which pertain to electromagnetic radiation, we address issues that are of particular interest when considering relativistic electrons encountered in a hot plasma such as the intergalactic medium. The discussion accommodates theoretical considerations of absorption theory which would have to be extended somewhat to address relativistic interactions involving paired secondary emission and absorption of electromagnetic radiation in scattering phenomena. This natural pairing of specific emission and absorption events relates closely to work by Lewis (1926), Wheeler and Feynman (1945), Cramer (1980 and 1986), and others who have shown that propagation of light may require an explicit pre-association of such emission and absorption events between material agents. Whereas observational aspects associated with this description are identical to those of more usual presentations of electromagnetic theory in the cases involving relatively stationary emitters and absorbers, observational aspects of relativistic aberration and Doppler effects associated with relatively moving intermediary 'observers' are characterized quite naturally in this approach as well.

Terminology

This appendix will shy away from much in the way of difficult mathematical prerequisites. However, equations will be presented wherever appropriate because there is much that can be inferred from an understanding of the symmetries of the equations and descriptions of the implied operations even by someone for whom the equations themselves may seem obtuse. Descriptions will be explicit – graphic where possible – but attempts have been made to avoid the more difficult aspects of the associated mathematics. Some minimal understanding of vector products and *divergence* and *curl* differential operations on a vector is essential to an understanding of the vector approach to electromagnetic field theory, of course. These definitions in Cartesian coordinates are as follows:

Inner or *dot* product: $\boldsymbol{U} \bullet \boldsymbol{V} \equiv U_x V_x + U_y V_y + U_z V_z$

Outer (*cross*) product: $\boldsymbol{U} \times \boldsymbol{V} \equiv \boldsymbol{i}\,(U_y V_z - U_z V_y) + \boldsymbol{j}\,(U_z V_x - U_x V_z) + \boldsymbol{k}\,(U_x V_y - U_y V_x)$

Gradient: $\nabla \alpha \equiv \boldsymbol{i}\,\partial\alpha/\partial \mathbf{x} + \boldsymbol{j}\,\partial\alpha/\partial \mathbf{y} + \boldsymbol{k}\,\partial\alpha/\partial \mathbf{z}$

Divergence: $\nabla \bullet \boldsymbol{U} \equiv \partial U_x/\partial \mathbf{x} + \partial U_y/\partial \mathbf{y} + \partial U_z/\partial \mathbf{z}$

Curl: $\nabla \times \boldsymbol{U} \equiv \boldsymbol{i}(\partial U_z/\partial \mathbf{y} - \partial U_y/\partial \mathbf{z}) + \boldsymbol{j}(\partial U_x/\partial \mathbf{z} - \partial U_z/\partial \mathbf{x}) + \boldsymbol{k}(\partial U_y/\partial \mathbf{x} - \partial U_x/\partial \mathbf{y})$

In the above definitions, ***U*** and ***V*** are vector fields; α, U_i's, and V_i's are scalars. The scalar U_x is the component of the vector ***U*** along the x axis. The *right-hand rule* (see figure A.1) states that if you use the fingers on your right hand to indicate the direction of rotation of ***U*** into ***V***, then the extended thumb will be in the direction of the vector *cross* product. In these definitions, ***U*** is a vector function of x, y, z, t. The *basis* vectors ***i, j, k*** are unit vectors in the directions of the x, y, z axes, respectively. The vector $\partial\, a(s)/\partial$ s is the partial derivative (the "slope" or rate of change) of the function *a(s)* with respect to the independent variable s. Scalars $\partial U_i/\partial$ s are the partial derivatives of scalar components of the vector ***U*** with respect to the independent variable s. The electric field ***E*** is, for example, the gradient of a scalar potential field. Note: Determining the *divergence* and *curl* of a vector is sufficient to determine the vector itself to within a vector constant throughout the region for which the relations apply.

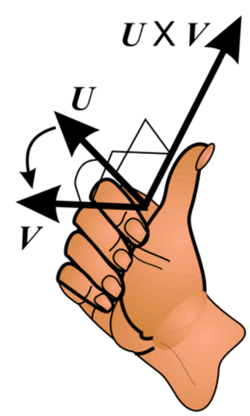

Figure A1: The right-hand vector cross product rule

Electromagnetic theory

In the interaction theory of radiation being discussed here, all of the overwhelming evidence of experimental confirmation of the theoretical origins of electromagnetic theory remain unchallenged and have intentionally not been altered. Maxwell's differential equations consolidate these results and are, therefore, accepted without change. They are:

1) Coulomb's law: *macroscopic* field – inhomogeneous equation

$$\nabla \bullet \boldsymbol{D} = \rho$$

2) Absence of monopoles: *microscopic* field – homogeneous equation

$$\nabla \bullet \boldsymbol{B} = 0$$

3) Faraday's law: *microscopic* fields – homogeneous equation

$$\nabla \times \boldsymbol{E} = -\,\partial \boldsymbol{B}/\partial \mathbf{t}$$

4) Amphere/Maxwell's law: *macroscopic* fields – inhomogeneous equation

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \partial \boldsymbol{D}/\partial \mathbf{t}$$

In these equations in *rationalized mks* units experimentally known vector functions ***D***, ***H***, ***E***, ***B***, and ***J*** are related one to another.[*] [See note at end of article.] The scalar function ρ is the *charge density* throughout the region for which the equations pertain. The vector quantity ***J*** is a characterization of the amount and direction of *conduction current* throughout the region. Boundary conditions of the region to which the equations are to pertain may further constrain the relationships among the various vector field quantities. The relationships define a nearly symmetric cycle; if ρ and ***J*** vanish throughout the region, all the equations become homogeneous differential equations of identical form and the symmetry is obviously complete. Since we will be dealing with the propagation of light in a vacuum between encounters, this symmetry will be assumed throughout the remainder of this article. Of the four remaining vector field quantities, two involve fields associated with electrical effects and two involve fields associated with magnetic effects. Two *constitutive* relation equations define and relate dual *microscopic* and *macroscopic* electric and magnetic fields as follows:

5) electrical: $\boldsymbol{D} = \varepsilon \boldsymbol{E}$ *macroscopic* field relation to *microscopic* field

6) magnetic: $\boldsymbol{H} = \mu^{-1} \boldsymbol{B}$ *macroscopic* field relation to *microscopic* field

where ε is the *permittivity* and μ the *permeability* of the medium. Both these quantities are typically scalars, but in certain media there are anisotropic distortion effects that can be characterized by a tensor representation of these quantities. These two equations reflect the fact that only one of the quantities (called the *microscopic* field – on the right) in each field category will be associated directly with emission; it is independent of the structural characteristics of interacting media throughout the region of consideration. The other two are *induced* in part by the *microscopic* fields and are called the *macroscopic* fields; these terms have more to do with *externality* of origination than with the *size* in electromagnetic theory. In a vacuum, the scalar constitutive coefficients are typically identified as ε_o and μ_o, whose values depend upon the system of units chosen. The speed of propagation of a wave function that satisfies Maxwell's equations will be seen to be determined by these quantities and in particular for propagation in a vacuum, that instantaneous speed will be:

7) speed of light in vacuum: $c \equiv (\mu_o\ \varepsilon_o)^{-1/2}$

In addition to Maxwell's equations, one must acknowledge the role of the *Lorentz force* on isolated charges as of extreme relevance to electrodynamics where there is relative motion of the charge in *microscopic* electromagnetic fields. It is given by:

8) Lorentz force: $\boldsymbol{L} = q\ (\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$ *microscopic* fields

where q is the scalar quantity of a specific charge that is in motion and $\boldsymbol{v}$ is the vector velocity of the charge relative to a test charge of unit magnitude experiencing the force. Thus the instantaneous electromotive force on a unit charge depends on magnetic as well as the usual electric forces in that case.

Deriving and solving radiation wave equations

Derivation of the wave equations from Maxwell's equations is problematical in several regards. Although there are two *microscopic* (2 and 3) and two *macroscopic* (1 and 4) equations, substitutions using *constitutive* relations (5 and 6) must be used to obtain the wave equations. The implications of the original four field equations, which seem clear, can easily be lost in the process of solution. For example, by these substitutions, solutions can be obtained for the *microscopic* fields $\boldsymbol{E}$ and $\boldsymbol{B}$ with the resulting equations looking *as though* they should be interpreted as the fluctuating electric and magnetic fields of an emitter *independent* of the medium or the ultimate absorber of the radiation. Here only the speed of propagation appears to be affected by the medium:

9) $\nabla^2 \boldsymbol{E} = -\ \mu\varepsilon\, \partial^2 \boldsymbol{E}/\partial t^2$

10) $\nabla^2 \boldsymbol{B} = -\ \mu\varepsilon\, \partial^2 \boldsymbol{B}/\partial t^2$

The definition of $\nabla^2 \boldsymbol{U}$ can be elaborated from the definitions above for the dot product of a gradient operator: $\nabla^2 \boldsymbol{U} \equiv \nabla \cdot \nabla \boldsymbol{U}$. The wave equations themselves derive from the vector identity $\nabla \times (\nabla \times \boldsymbol{U}) \equiv \nabla(\nabla \cdot \boldsymbol{U}) - \nabla^2 \boldsymbol{U}$ and by substitutions from constitutive relations into Maxwell's equations. The wave equations 9) and 10) each derive directly from Maxwell's equations 3) and 4) in addition to either 1) or 2) with constitutive relation substitutions occurring twice in the process. So these are hardly isolated conditions applicable solely to an emitter.
These equations describe propagational wave phenomena. In general solutions will be complex quantities, only the real parts of which are of any interest experimentally. Solutions shown in figure A2 are of the form:

11) $\boldsymbol{E} = \boldsymbol{E_o}\, e^{\pm i (\kappa \cdot r - i\, \omega t)}$

12) $\boldsymbol{B} = \boldsymbol{B_o}\, e^{\pm i (\kappa \cdot r - i\, \omega t)}$

$\boldsymbol{E_o}$ and $\boldsymbol{B_o}$ are constant vectors for plane polarized waves. Substitution back into Maxwell's *divergence* equations results in further constraints on $\boldsymbol{E}$ and $\boldsymbol{B}$ such that both must be perpendicular to the direction of propagation given by the *wave vector* κ, whose magnitude is given by $\kappa = (\mu\ \varepsilon)^{1/2}\omega$, where ω is the *angular frequency* of the radiation. This constraint is the basis of the notable *transverse* wave nature of light. Substituting into Maxwell's *curl* equations places additional constraints on E and B such that they must always be in phase and of equal in magnitude in addition to being at right angles to each other. By superposition of linearly independent solutions with uniquely paired $\boldsymbol{E_o}$ and $\boldsymbol{B_o}$ values, one obtains the more general elliptical polarization solutions – plane and circular polarization being the special cases shown in figure A2.

Are there preferred solutions to Maxwell's equations?

It is apparent that Maxwell's equations may be used to determine valid solutions for all four of the fields. But which wave equations (if any) *inherently couple* as a single transverse wave? In other words, do $\boldsymbol{E}$ and $\boldsymbol{B}$, $\boldsymbol{E}$ and $\boldsymbol{H}$, $\boldsymbol{D}$ and $\boldsymbol{H}$, or $\boldsymbol{D}$ and $\boldsymbol{B}$ constitute the most meaningful description of the radiation we associate with these equations? With such a plethora of possibilities, which (if any) of these solutions should be preferred?

In consideration of these questions, we note that radiation energy density and energy flow (as electromagnetic *momentum*) equations both involve equally coupled *microscopic* and *macroscopic* fields for each as follows:

13) energy density: $u = ½ (\boldsymbol{E} \bullet \boldsymbol{D} + \boldsymbol{B} \bullet \boldsymbol{H})$

14) energy flow: $\boldsymbol{P} = \boldsymbol{E} \times \boldsymbol{H}$

More than any other single equation, the latter Poynting vector equation symbolizes the *transverse* nature of electromagnetic radiation (refer to the *right-hand rule* above for an intuitive feel for this quantity) that distinguishes it from *longitudinal* vibrations characteristic of sound propagation. Furthermore, *this* equation clearly indicates equal participation by *macroscopic* fields associated within the medium and/or absorption. With only an emitting and an absorbing atom under consideration, *E* would clearly be associated with the emitter, *H* with the absorber. Thus, energy and momentum considerations would seem to suggest that ***E*** and ***H*** occupy preeminent positions, as the fields most naturally characterizing radiative energy transfer.

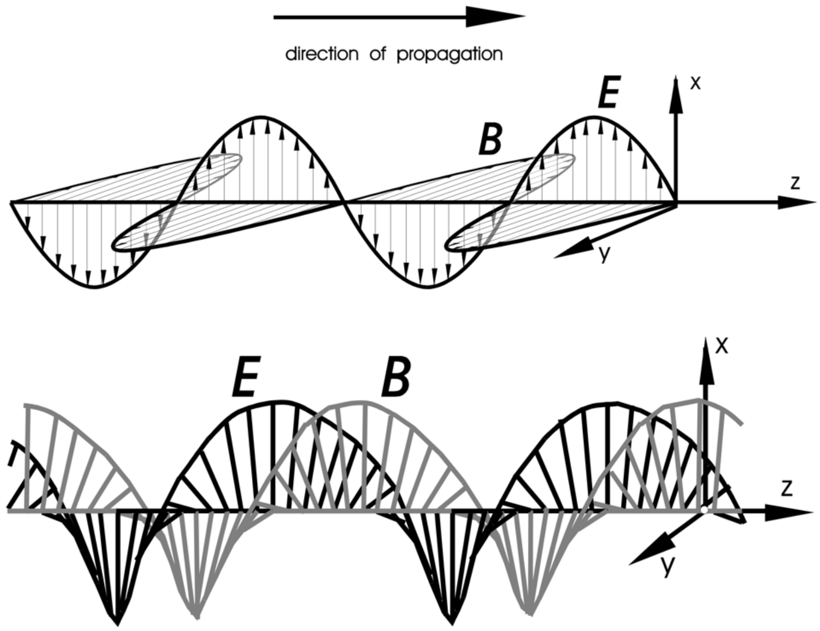

Figure A2: Plane and circularly polarized solutions of Maxwell's homogeneous differential equations

Proponents of absorption theory have advocated an equal role for absorption to the one usually associated exclusively with emission. They have pointed out that, in addition to field alternatives, there are two sets of valid solutions to *whichever* set of wave equations are selected. One of these alternatives – identified as the *retarded potential* solution (associated with propagation from the emitter toward the absorber) – has been the traditionally selected solution to Maxwell's equations. The other allowed solution identified as the *advanced potential* solution (associated with propagation from the absorber toward the emitter) was subsequently proposed as being equally legitimate by Wheeler and Feynman (1945). Naturally the *retarded* solution was exclusively in vogue until absorption theory was seriously considered, the *advanced* solution having always seemed to correspond to the *non-physical* situations of a signal *arriving* at the moment that emission occurs as though by divine intervention. More recently Cramer has proposed a similar reinstatement to vitalize a "transaction interpretation" of quantum mechanics. He demonstrates the role of the two waves as illustrated in figure A3 taken from his presentations (1986, p. 659).

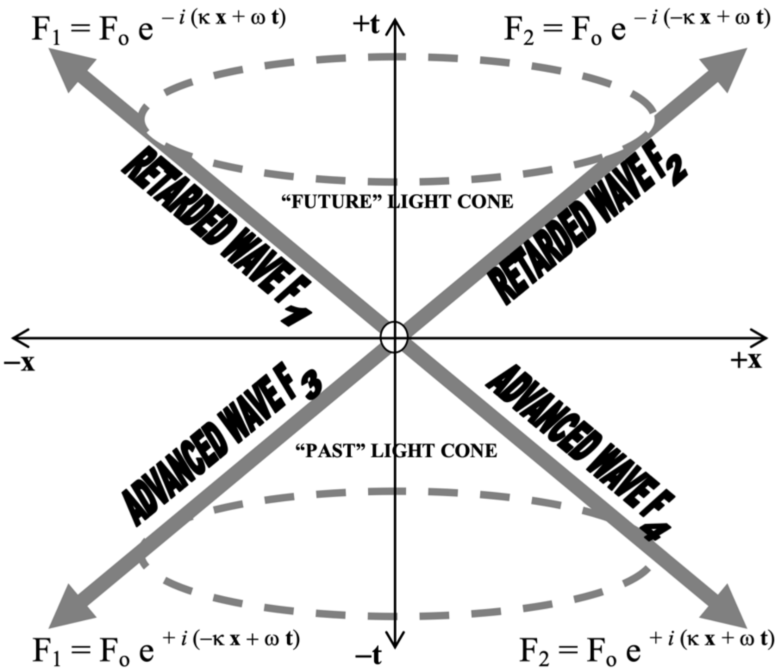

Figure A3: Minkowski spacetime diagram showing the propagation of advanced and retarded waves from an emission locus at (x,t)=(0,0)

Here there is an *arithmetic* assignment of plus and minus signs to be associated with advanced and retarded waves, but nothing that could be considered a *physical* assignment specific to the roles of emission and absorption so clearly integral to this whole process. None of these early investigators addressed the more obviously physical allocation of fields specific to material entities associated with the emission and absorption of the radiation. The assignments fit naturally into this scheme.

But the conclusion that redundant sets of solutions are involved equally in the transaction is a conclusion that absorption theorists have long maintained, advocating acceptance of both the plus and minus signs in the exponential expression of the wave solutions provided in the equations 11) and 12). This author is convinced that the respective microscopic and macroscopic physical fields should also be acknowledged as being uniquely associated with these four solutions as well rather than merely including solutions with an arbitrary alternation of arithmetic sign in an attempt to restore physically meaningful interpretations to the two solutions. There is obviously much more to it than that.

This reluctance to make distinctions between the frame of reference of the fields is no doubt an outgrowth of the frame independence that has resulted from Einstein's *law of the transmission of light* for which it should make no difference in which frame the source of the emission and the absorber of the radiation happen to reside. Thus, the early investigators did not allocate *macroscopic* fields associated specifically with *absorption* or the *microscopic* ones with *emission* as seems only reasonable to this author. Nor did they attempt to exploit complimentary symmetries among the fields, which would seem so natural to that endeavor. If we had solved Maxwell's equations for *H* and *D* instead of *E* and *B*, for example, we might in effect have solved for what could be called an *absorber wave equation* as against an *emitter wave equation.* For reasons cited above and others beyond the scope of the current effort, the author believes neither of these to be precisely valid designations, however. There is in either case an interaction between the microscopic and macroscopic fields to be taken into account. Perhaps we are at least discovering why *four*, seemingly redundant, rather than just *two* such field vectors have been required to fully determine electromagnetic transactions even in a vacuum.

Of course, when dealing with a relatively stationary emitter and absorber there would be no measurable difference, but in dynamic situations epistemological differences abound. These differences derive from directional distortions associated with relativistic aberration. But again, further discussion of this topic is beyond the bounds of the current Appendix and may be found in Bonn (2008).

NOTE:

*The quantum theory of light does not substantially alter the results of Maxwell's approach that was historically significant to the development of relativity and so we will go with that more intuitive approach. This is in accordance with decisions by Wheeler and Feynman, as well as Cramer cited above in their similarly motivated analyses. The fashionable geometrical approach using generic differentiation of an electromagnetic field strength tensor to represent these equations, while economical in terminology, de-emphasizes the complimentary nature of emission and absorption processes envisioned here, since typically the tensor has been deployed with exclusively microscopic fields.